MINIMUM COST DESIGN OF PRE-CAST POST-TENSIONED PRESTRESSED CONCRETE BRIDGE DECKS

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CERTIFICATE

This is to certify that this work entitled 'Minimum Cost Design of Pre-cast Post-tensioned Prestressed Concrete Bridge Decks' has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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CONTENTS

ABSTRACT

In this thesis a two lane bridge made of prestressed concrete has been optimized for minimum cost. The cost of bridge deck consists of the cost of longitudinal and transverse beams. The analysis of the deck has been via equivalent orthotropic plate and individual members forming the deck have been designed by working load method for loads and stresses specified by Indian Roads Congress.

In this thesis two commonly used sections in the bridge decks have been taken up. The sections are T-section and Hollow Box section. The non-linear programming problem of finding the minimum cost design has been solved by sequential unconstrained minimization technique using interior penalty function. Davidon-Fletcher-Powell algorithm has been used for unconstrained minimization, while Fibonacci method is employed for linear search.

Graphs has been generated to help in calculating the maximum transverse moment in bridge deck for few classes of loadings of Indian Roads Congress. A parametric study has been performed with number of longitudinal beams in a deck changing from 5 to 9. The spans considered are 24m and 30m.

CHAPTER I

INTRODUCTION

1.1 Introduction

Simply supported prestressed concrete bridges for highways consist of a number of longitudinal beams which are hinged to the support at one end and resting on rollers at the other. These longitudinal beams are made monolithic with the transverse beams which are also of prestressed concrete. The number of transverse beams are usually predetermined. With such a configuration of beams in a bridge deck, it can be idealized as a gridwork. The analysis of gridwork subject to transverse eccentric loads by transforming it to an equivalent orthotropic plate was pioneered by Guyon [1]. It was further developed by Massonnet [2]. The formal technique of analysis and design of such a gridwork via equivalent orthotropic plate was fully documented by Rowe [3]. Even today it is a classical approach for Bridge Designers.

For a developing country like India where the transportation system is yet to be perfected, highways and hence bridges will assume great importance. Prestressing techniques have been understood well by now. A number of small factories exist throughout the country where pre-cast sections can be made and transported in convenient sizes to the site and then post-tensioned in situ. In the present work few commonly

used sections for pre-cast and post-tensioned concrete bridge decks have been optimized for minimum cost. The loads acting on the bridge are as per the criteria laid down by Indian Roads Congress (IRC)[5]. The working load method has been adopted for design. Optimum design is arrived at by sequentia unconstrained minimization technique [6] using interior penalty function. The Davidon-Fletcher-Powell algorithm has been used for unconstrained minimization and Fibonacci method is employed for linear search.

1.2 Summary of Previous Work

There has been sufficient published work on the analysis of bridge decks [3,4,7]. However, little work on optimum design of bridge decks is available. A completely automated design of a three span bridge of prestressed concrete has been developed by Sawko and Willcock [8] to satisfy the loading requirements of British Standard.

The handbook [9] prepared by Structural Engineering Research Centre (SERC) gives optimum standard sections for the loading proposed by IRC. The sections considered are inverted T-beam, Hollow Box, composite I section and T section for the spans ranging from 12m to 36m at an interval 1.5m. The clear carriage way taken is 7.34m. The number of beams considered in case of T-section bridge is 5, 6, 7, 8 and 9. The cost of these bridges is compared. A bridge

having five beams turn out to be of minimum cost. In the case of Hollow Box section bridge, the number of beams considered is 9 and 12. The cost of the Hollow Box bridge having nine beams turns out to be the minimum. While calculating the cost of bridge deck the cost of prestressing cables in transverse beams are not taken into account. Moreover the minimum cost is arrived at by a brute force method of comparing the cost of bridges with beams increasing in depth by one centimeter. The distribution coefficients for longitudinal moments corresponding to unit point load are read from graphs given in reference [3] for various standard reference points on the deck.

The optimum design of T-beam bridges subject to 70R loading of IRC was carried out by Raman et al [10] satisfying the stress condition of IS-1343. The nonlinear optimum design problem has been solved by linearizing the functions through Taylor series. The method used for distribution of longitudinal moment is due to Courbon [11], which is a very approximate method.

Kumarasamy [12] minimized the total cost of a T-beam bridge having several simply supported spans using sequential linear programming technique. The cost function was minimized subject to constraints stipulated by IRC. The variables in the optimum design problem was number of beams, number of bays, number of prestressing cables besides the other

variables which define the geometry of the section. Here also the cost of transverse beams was completely neglected. The coefficients of distribution of longitudinal moment corresponding to unit point loads was supplied at discrete points. The graphs have been drawn to directly get the coefficients for distribution of longitudinal moments for two lane bridge for 70R, class AA and class A loads.

The T-beam sections used in prestressed concrete bridges were optimized by Kapoor and Bandhopadhyay [13] for a simple loading. The working load method, ultimate load method and reliability based method were used to optimize the cost of the section. It was shown that reliability based method gives the least cost design. However, the investigation suffers with the drawback that practical loadings have not been considered.

1.3 The Present Work

A two lane bridge with foot path on both sides has been optimized to obtain the minimum cost of the entire deck system. The following two commonly used sections in bridge decks have been chosen for study in the present work.

(i) T-section

(ii) Hollow Box section

The cost includes the cost of longitudinal beams, transverse beams and the prestressing cables. Equivalent

orthotropic plate analysis is used to determine the worst loaded beam in both longitudinal and transverse directions. All the longitudinal beams are of same cross-sectional area, shape and size. The transverse beams are also identical in detail.

A brief description of orthotropic plate analysis of bridge decks is given in Chapter II. The graphs developed for coefficient of distribution of transverse moments are also presented in this chapter. The graphs correspond to 70R and class AA loading placed to cause maximum moment in the transverse beams.

The working load method of design is described in brief in Chapter III. The formulation of the optimum design problem and the solution technique is also described in short in this chapter.

In the last chapter the results of two problems, one for each type of section, are presented. It is followed by discussion of the results.

It is concluded that T-section bridge deck is economical in comparison to Hollow Box section bridge decks for the spans studied. The parametric study on these bridge decks with increasing number of longitudinal beams reveal that the cost of bridge deck increases with increase of longitudinal beams. The use of 7 number of longitudinal beams

is recommended as the slight increase in cost of deck is expected to be wel-compensated in handling the reduced weight of the modules.

CHAPTER II

ANALYSIS OF BRIDGE DECKS

2.1 Introduction

There are various methods available for analysis of bridge decks [3,4,7,11]. The method adopted in the present work is the one which converts the gridwork of bridge decks to an equivalent orthotropic plate. This approach seems to be commonly used by designers. It uses a single set of distribution coefficients for two extreme cases of a no-torsion grillage and a full torsion slab to find the distribution of behavior parameter e.g. forces and deflections etc at any point of any type of the bridge. The governing differential equation of flexure for orthotropic plate and the boundary conditions,the equivalent orthotropic plate has to satisfy, is given in the following section. The different parameters involved in the solution of the governing differential equations are discussed alongwith definitions of coefficient for distribution of longitudinal and transverse moments. The procedure of finding the design moment for the longitudinal beams as well as the transverse beams are discussed in the subsequent sections. The chapter ends with the section describing the graphs generated in the present work to help in calculating the transverse moments.

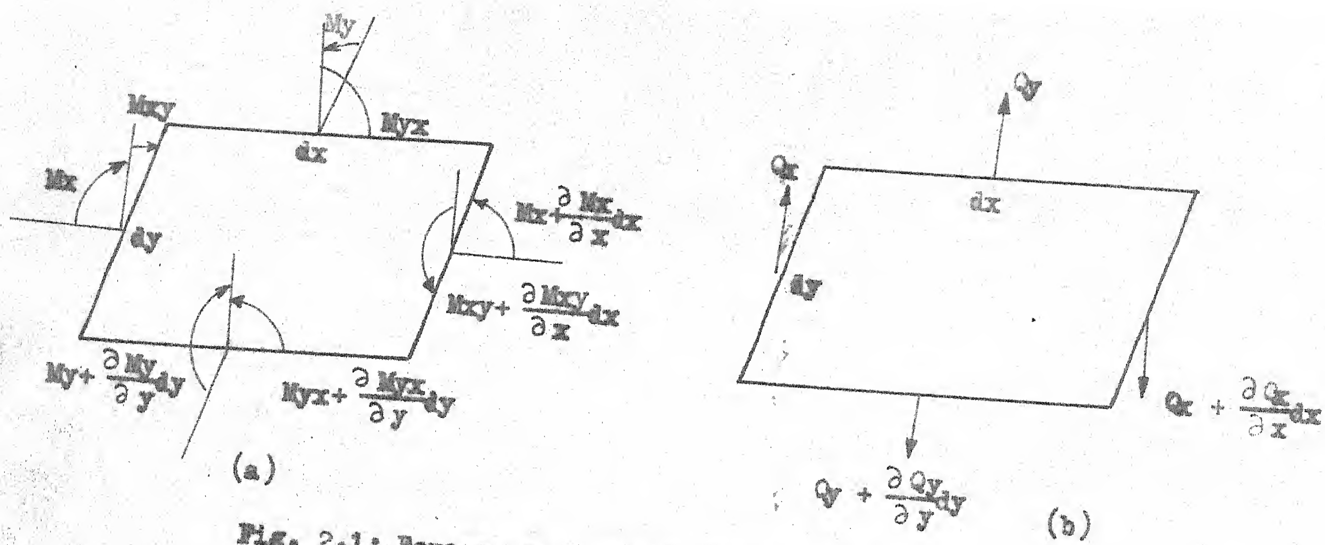


Fig. 2.1: Forces and moments on plate element

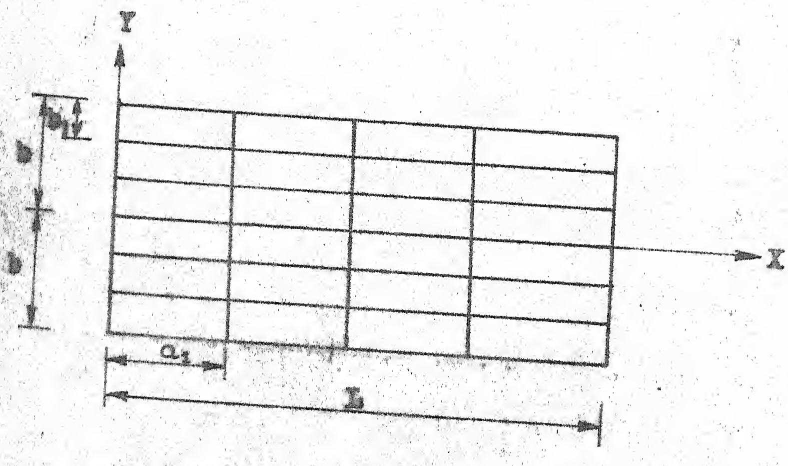


Fig. 2.2: Beam Grillage

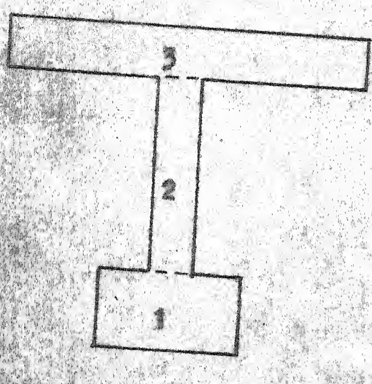


Fig. 2.3: Idealized T-section

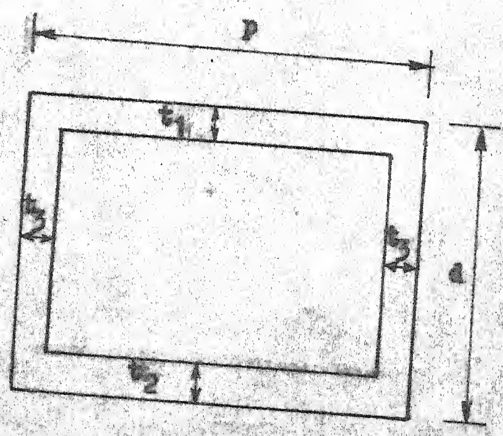


Fig. 2.4: Idealized Hollow Box Section

2.2 Governing Equations of an Orthotropic Plate

The theory of orthotropic plate is described in detail in reference 14. For completeness of presentation the equations in brief are given:

The stress-strain relationship is given by

$$\sigma_x = E'_x \varepsilon_x + E'' \varepsilon_y$$

$$\sigma_y = E'_y \varepsilon_y + E'' \varepsilon_x \qquad 2.1$$

$$\tau_{xy} = G\gamma_{xy}$$

where $E'' = \nu_x E'_y = \nu_y E'_x$: $E'_x = \dfrac{E_x}{1-\nu_x\nu_y}$; $E'_y = \dfrac{E_y}{1-\nu_x\nu_y}$ 2.2

assuming plane sections remain plane after bending, the strains in terms of deflection w(x,y) are given by

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}\ ; \quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}\ ; \quad \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x\partial y} \qquad 2.3$$

The corresponding stress components are

$$\sigma_x = -z\left(E'_x\frac{\partial^2 w}{\partial x^2} + E''\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -z\left(E'_y\frac{\partial^2 w}{\partial y^2} + E''\frac{\partial^2 w}{\partial x^2}\right) \qquad 2.4$$

$$\tau_{xy} = -2Gz\frac{\partial^2 w}{\partial x\partial y}$$

The bending and twisting moments are given by

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z\, dz = -\left(Dx\frac{\partial^2 w}{\partial x^2} + D_1\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z \, dz = -(Dy \frac{\partial^2 w}{\partial y^2} + D_1 \frac{\partial^2 w}{\partial x^2}) \qquad ...2.5$$

$$\text{and } M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z \, dz = 2Dxy \frac{\partial^2 w}{\partial x \partial y}$$

$$\text{where } \quad Dx = \frac{E'_x h^3}{12} \; ; \quad Dy = \frac{E'_y h^3}{12} \; ; \quad D_1 = \frac{E'' h^3}{12} \; ; \quad Dxy = \frac{Gh^3}{12} \qquad ...2.6$$

By using moment equilibrium of the element, the relationship for shear forces Qx and Qy is given as

$$Qx = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} = -\frac{\partial}{\partial x}(Dx \frac{\partial^2 w}{\partial x^2} + H\frac{\partial^2 w}{\partial y^2})$$

$$Qy = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} = -\frac{\partial}{\partial y}(Dy \frac{\partial^2 w}{\partial y^2} + H\frac{\partial^2 w}{\partial x^2}) \qquad ...2.7$$

$$\text{where } \quad H = D_1 + 2D_{xy} \qquad ...2.8$$

By considering the static equilibrium of vertical forces acting on the element shown in Fig. 2.1, we get

$$Dx \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + Dy \frac{\partial^4 w}{\partial y^4} = q \qquad ...2.9$$

The equation (2.9) is the governing differential equation for flexure of orthotropic plate where H is a measure of torsional rigidity of the plate.

2.3 Bridge Deck and Equivalent Orthotropic Plate

A bridge deck has equally spaced beams in both longitudinal and transverse directions. In general, the stiffness and spacings of these beams are different in different directions.

Thus if the bridge deck has to be transformed to an orthotropic plate, the elastic parameters relating to the substitute system are to be assumed as continuously distributed and their respective values are to be defined per unit length.

Consider a grillage consisting of two systems of equally spaced parallel beams and loaded perpendicularly to x-y plane as shown in Figure 2.2. If I and J are flexural moments of inertia of individual beams along x-axis (longitudinal direction) and y-axis (transverse direction) respectively. Then

$$Dx = \frac{EI}{b_1} \; ; \quad Dy = \frac{EJ}{a_1} \qquad \ldots 2.10$$

where E is young's modulus, a_1 and b_1 are spacing of beams in x and y directions respectively. Thus Dx and Dy represent flexural rigidity of equivalent system for bending in x and y directions respectively.

Again if I_o and J_o are torsional moments of inertia of individual beams along x-axis and y-axis respectively,

$$\gamma_x = \frac{GI_o}{b_1} \; ; \quad \gamma_y = \frac{GJ_o}{a_1} \qquad \ldots 2.11$$

γ_x and γ_y are torsional rigidity in x and y directions respectively.

The bending and twisting moments can now be written as

$$M_x = - Dx\frac{\partial^2 w}{\partial x^2} \; ; \quad M_y = - Dy\frac{\partial^2 w}{\partial y^2} \; ; \quad M_{xy} = \gamma_x \frac{\partial^2 w}{\partial x \partial y}$$

$$\text{and} \quad M_{yx} = -\gamma_y \frac{\partial^2 w}{\partial x \partial y} \qquad \ldots 2.12$$

Considering the vertical equilibrium of an element, the governing differential equation of flexure comes out as

$$Dx \frac{\partial^4 w}{\partial x^4} + (\gamma_x + \gamma_y)\frac{\partial^4 w}{\partial x^2 \partial y^2} + Dy \frac{\partial^4 w}{\partial y^4} = q \qquad ...2.13$$

or $$Dx \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + Dy \frac{\partial^4 w}{\partial y^4} = q \qquad ...2.14$$

where $2H = \gamma_x + \gamma_y$ ...2.15

and it represents the measure of torsional rigidity of the equivalent orthotropic plate.

The equations (2.9) and (2.14) are of same mathematical form. If Poisson's ratio ($\nu = 0.15$ for concrete) is neglected the expressions for M_x, M_y are same for equivalent orthotropic plate and the true orthotropic plate. If Qy is defined as

$$Qy = - Dy\frac{\partial^3 w}{\partial y^3} - \tfrac{1}{2}(\gamma_x + \gamma_y)\frac{\partial^3 w}{\partial x^2 \partial y} \qquad ...2.16$$

the expression for M_{xy} and M_{yx} are also identical for the two systems and the value of D_1 approximately becomes zero.

By comparing equations (2.15) and (2.8)

$$\gamma_x + \gamma_y = 2H = 4Dxy \qquad ...2.17$$

2.4 Boundary Conditions

The bridge is simply supported along the two opposite edges at $x = 0$ and $x = L$, and is free at the remaining two edges $y = \pm b$.

At the supported edges the deflection is obviously zero for non-sinking supports i.e.

$$w = 0 \text{ at } x = 0 \text{ and } L$$

and the bending moment $M_x = 0$, i.e. ...2.18

$$\frac{\partial^2 w}{\partial x^2} = 0 \text{ at } x = 0 \text{ and } L$$

Along the free edges bending moments M_y is zero, i.e.

$$\frac{\partial^2 w}{\partial y^2} = 0 \text{ at } y = \pm b$$

and the shear is also zero i.e.

$$Qy = Dy \frac{\partial^2 w}{\partial y^3} + 2Dxy \frac{\partial^3 w}{\partial x^2 \partial y} = 0 \; ; \quad \text{at } y = \pm b \qquad ...2.19$$

2.5 Dimensionless Parameters

In all practical cases the value of torsional rigidity given by the sum $(\gamma_x + \gamma_y)$ is enclosed between two limits corresponding to the simple grid of beams on the one hand and the isotropic slab on the other.

Putting

$$\gamma_x + \gamma_y = 2\alpha \sqrt{DxDy} \qquad ...2.20$$

the equation of elastic surface (in terms of deflection w) in the simplified form is

$$Dx \frac{\partial^4 w}{\partial x^4} + 2\alpha\sqrt{DxDy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + Dy \frac{\partial^4 w}{\partial y^4} = q \qquad ...2.21$$

and for the shear force Qy

$$Qy = - Dy \frac{\partial^3 w}{\partial y^3} - \alpha\sqrt{DxDy} \frac{\partial^3 w}{\partial x^2 \partial y} \qquad ...2.22$$

In the last two equations the dimensionless parameter α is characteristic for the resistance in torsion. The range of variation of α is limited by the values 0 and 1. For $\alpha = 0$ the system reverts back to a very simple grid of beams with no torsional rigidity while for isotropic plate $\alpha = 1$. From Equation (2.20) we have

$$\alpha = \frac{\gamma_x + \gamma_y}{2\sqrt{DxDy}} \qquad \text{...2.23}$$

Equation (2.23) can also be written as follows:

$$\alpha = \frac{\gamma_x + \gamma_y}{2\sqrt{DxDy}} = \frac{G(i_o + j_o)}{2E\sqrt{ij}} \qquad \text{...2.24}$$

where

$i_o = \frac{I_o}{b_1}$ = Torsional inertia per unit width of bridge

$j_o = \frac{J_o}{a_1}$ = Torsional inertia per unit length of bridge

$i = \frac{I}{b_1}$ = Moment of inertia per unit width of bridge

$j = \frac{J}{a_1}$ = Moment of inertia per unit length of bridge

The determination of I_o and J_o is a tedious job. For a T-section which consists of three flat plate elements, Figure 2.3, I_o or J_o is given in reference 3

$$I_o \text{ or } J_o = k_1(2a_1)^3 \cdot 2b_1 + k_2(2a_2)^3 \cdot 2b_2 + \tfrac{1}{2}k_3(2a_3)^3 2b_3 \qquad \text{...2.25}$$

where suffixes refer to rectangle 1, 2 and 3. The value of b corresponds to the longer side of the rectangles. The torsional constants k used in Equation (2.25) are given by [15]

$$k_i = \frac{1}{3} - \frac{64}{\pi^5} \frac{a_i}{b_i} \sum_{m=1,3,5..}^{\infty} \frac{1}{m^5} \tanh \frac{m\pi b_i}{2a_i} \qquad ...2.26$$

For a Hollow Box section shown in Fig. 2.4.

$$I_o \text{ or } J_o = \frac{4A^2}{\frac{p-2t_3}{t_1} + \frac{p-2t_3}{t_2} + \frac{2(d-t_1-t_2)}{t_3}} \qquad ...2.27$$

where A is area of whole in the section. Guyon [1] in his work on grid system with no torsional rigidity introduced another dimensionless parameter θ, defining as

$$\theta = \frac{b}{L} \sqrt[4]{\frac{Dx}{Dy}} \qquad ...2.28$$

θ is called the flexural parameter since it gives a measure of flexibility of the deck system. It can also be written in simplified form as

$$\theta = \frac{b}{L} \sqrt[4]{\frac{i}{j}} \qquad ...2.29$$

The width of the equivalent orthotropic plate turns out to be equal to the width of the bridge, because all the longitudinal beams considered in the present work are identical which generally happens to be the case in all practical designs.

α and θ given by equations (2.24) and (2.29) respectively

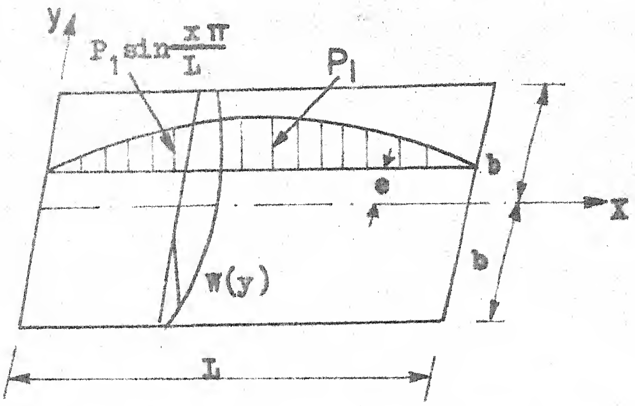
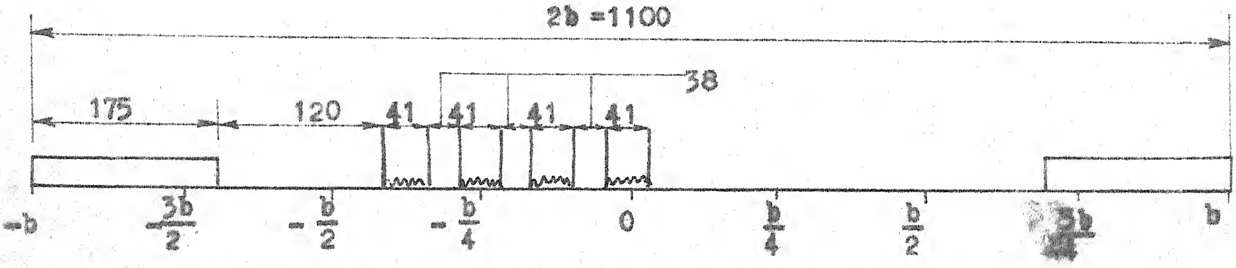
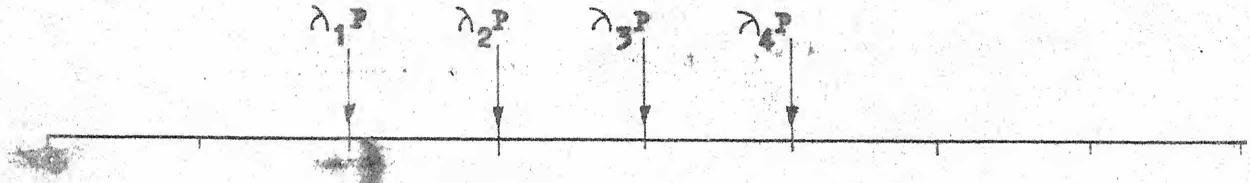


Fig. 2.5: Eccentric sinusoidal load on bridge deck

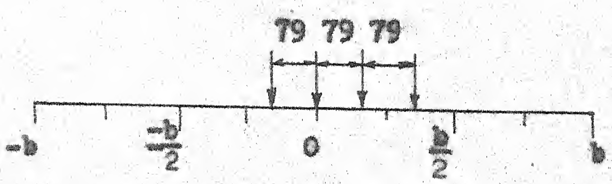


(a) ACTUAL Transverse position of wheels of 70R Loads

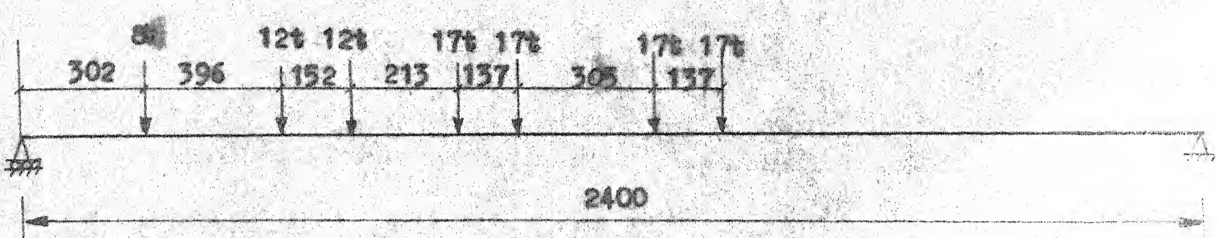


(b) Equivalent loads at reference station

Fig. 2.6: Transverse position of 70R load for worst effect



(a) Transverse position of wheels



(a) Longitudinal position of wheels

are for homogeneous cross-sections. In case of prestressed concrete sections where no tensile stresses are allowed, the cross-sections can be considered to be homogeneous. IRC does not permit any tensile stresses to develop in these sections. Hence the sections considered in the present work are homogeneous in this context.

2.6 Principal Coefficient of Distribution of Longitudinal Moments

The load acting on the equivalent orthotropic plate is assumed to have a sinusoidal variation along the span of bridge, or

$$p(x) = p_1 \sin \frac{\pi x}{L} \qquad \ldots 2.30$$

and the load is assumed to be acting at an eccentricity 'e' (Fig. 2.5) in the y-direction. Due to this load the deflection can be assumed as

$$w(x,y) = W(y) \sin \frac{\pi x}{L} \qquad \ldots 2.31$$

If the load defined by equation (2.30) is distributed over the entire width 2b of the system, the intensity of this load, uniform in the y-direction becomes

$$p_o(x) = p_o \sin \frac{\pi x}{L} \qquad \text{where } p_o = \frac{p_1}{2b}$$

and deflection is of the form

$$w_o(x) = W_o \sin \frac{\pi x}{L} \qquad \ldots 2.32$$

The ratio of the two deflections, as produced by the line-load p_1 and by the distributed load p_o respectively is called the principal coefficient of distribution of longitudinal moments, i.e.

$$K(y) = \frac{w(x,y)}{w_o(y)} = \frac{W(y)}{W_o} \qquad \ldots 2.33$$

The value of K(y) will depend on the parameters θ and α, relative eccentricity e/b, and ratio y/b the relative ordinate of the section considered. Moreover, K(y) gives the deflection at a point y/b for a unit sinusoidal load acting at an eccentricity e/b.

Massonnet [16] has solved the equation 2.14 for a load $p(x) = p_1 \sin \frac{\pi x}{L}$. The system analysed is a grid with no torsional resistance. The deflected surface is given as

$$w = \frac{p_1 \lambda L^4}{Dx\pi^4} \sin \frac{\pi x}{L} \frac{1}{\mathrm{Sinh}^2 2\lambda b - \sin^2 2\lambda b} \Big(2\cosh\lambda(b+y) \cos\lambda(b+y)$$

$$[\sinh 2\lambda b \cos\lambda(b+e) \cosh\lambda(b-e) - \sin 2\lambda b \cosh\lambda(b+e) \cdot$$

$$\cos\lambda(b-e)] + [\cosh\lambda(b+y) \sin\lambda(b+y) + \sinh\lambda(b+y) \cos\lambda(b+y)]$$

$$\{\sinh 2\lambda b [\sin(b+e) \cosh\lambda(b-e) - \cos\lambda(b+e) \sinh\lambda(b-e)]$$

$$+ \sin 2\lambda b [\sinh\lambda(b+e) \cos\lambda(b-e) - \cosh\lambda(b+e) \sin\lambda(b-e)\}\Big)$$

$$\ldots 2.34$$

where $\lambda = \frac{\pi}{L\sqrt{2}} \sqrt[4]{\frac{Dx}{Dy}} = \frac{\pi\theta}{b\sqrt{2}}$ ...2.35

If the load $p_1 \sin \frac{\pi x}{L}$ is uniformly distributed over the entire width 2b the system will deflect to a cylinder as

given by equation 2.37. By dividing equation 2.39 by 2.37 the principal coefficient of lateral distribution is obtained as

$$K_o = 2\lambda b \frac{1}{\sinh^2 2\lambda b - \sin^2 2\lambda b}\Big(2\cosh\lambda(b+y)\cos\lambda(b+y)\,[\sinh 2\lambda b\,\cdot$$

$$\cos\lambda(b+e)\cosh\lambda(b-e) - \sin 2\lambda b\cosh\lambda(b+e)\cos\lambda(b-e)]$$

$$+\,[\cosh\lambda(b+y)\sin\lambda(b+y) + \sinh\lambda(b+y)\cos\lambda(b+y)]$$

$$\{\sinh 2\lambda b[\sin\lambda(b+e)\cosh\lambda(b-e) - \cos\lambda(b+e)\sinh\lambda(b-e)]$$

$$+\,\sin 2\lambda b[\sinh\lambda(b+e)\cos\lambda(b-e) - \cosh\lambda(b+e)\sin\lambda(b-e)]\}\Big) \quad \ldots 2.36$$

The subscript 'o' denotes that the formula holds true for $\alpha = 0$. The equation (2.36) is valid for the portion of the width of the bridge where the ordinate y is less than the eccentricity, e, of the load. In case we are on a point in the bridge where y is greater than e, the sign of y and e in the above equation get changed.

For $\alpha = 1$ the corresponding coefficient K_1 can be calculated using the formula of Guyon [1] given as

$$K_1 = \frac{\sigma}{2\sinh^2\sigma}\Big[(\sigma\cosh\sigma + \sinh\sigma)\cosh\theta\chi - \theta\chi\sinh\sigma\sinh\theta\chi$$

$$+ \frac{R_\varphi R_\psi}{3\sinh\sigma\cosh\sigma - \sigma} + \frac{Q_\varphi Q_\psi}{3\sinh\sigma\cosh\sigma + \sigma}\Big] \quad \ldots 2.37$$

where $R_\varphi = (\sigma\cosh\sigma - \sinh\sigma)\cosh\theta\varphi - \theta\varphi\sinh\sigma\sinh\theta\varphi$

$$R_\psi = (\sigma\cosh\sigma - \sinh\sigma)\cosh\theta\psi - \theta\psi\sinh\sigma\sinh\theta\psi \quad \ldots 2.38(a)$$

$$Q_\varphi = (2\sinh\sigma + \sigma\cosh\sigma)\sinh\theta\varphi - \theta\varphi\sinh\sigma\cosh\theta\varphi$$

$$Q_\psi = (2\sinh\sigma + \sigma\cosh\sigma)\sinh\theta\psi - \theta\psi\sinh\sigma\ \cosh\theta\psi$$

$$\phi = \frac{\pi y}{b} \qquad \psi = \frac{\pi e}{b} \qquad \ldots 2.38(b)$$

$$\sigma = \theta\pi \qquad \chi = \pi - |\varphi - \psi|$$

For a practical case the value of α lies between zero and one. So to find K_α an interpolation formula

$$K_\alpha = K_o + (K_1 - K_0)\sqrt{\alpha} \qquad \ldots 2.39$$

is used. The value of K_α given by equation (2.39) is increased by 10 percent to make the theoretical results correspond to actual experimental results [3].

For a bridge of known dimensions θ and α can be easily calculated. K_o and K_1 can be found for a given value of θ and for different values of eccentricity of load, e/b ranging from -b to b at intervals of b/4, and for different discrete locations on the bridge, y/b ranging from o to b at intervals of b/4. These values of K_o and K_1 are stored in two (5 x 9) matrices. Using Maxwell's theorem the two (5 x 9) matrices can be expanded to (9 x 9) matrices to include the effect at other points of the bridge corresponding to y/b ranging from -b to o at intervals of b/4. The nine discrete points $(-b, -\frac{3b}{4}, -\frac{b}{2}, -\frac{b}{4}, o, \frac{b}{4}, \frac{b}{2}, \frac{3b}{4}, b)$ are hence forth referred to as reference stations.

2.7 Maximum Longitudinal Moments

Maximum longitudinal moment can be calculated from the two matrices $[K_o]$ and $[K_1]$ for any practical case, using the interpolation formula given by equation (2.39).

For the spans under consideration (24m and 30m) it has been established [12] that 70R loading of IRC gives maximum longitudinal moments. The maximum eccentricity of load is at a distance of 1.2 metre from the edge of the foot path. The actual load position for worst effect is shown in Fig. 2.6. As the wheel loads do not coincide with the reference stations, so these loads are replaced by equivalent loads at the reference stations. This is done to facilitate the use of matrices $[K_o]$ and $[K_1]$ which have been generated for these reference stations. The equivalent loads at the standard reference stations are the reactions of the simple beam of span b/4 [3]. The loads at the reference stations are thus $\lambda_1 P$, $\lambda_2 P$, $\lambda_3 P$, $\lambda_4 P$ where $\sum_{n=1}^{4} \lambda_n = 4.0$. λ's are known as weighting factors. The maximum element of the vector obtained by

$$\frac{1}{4}[\lambda]^T[K_o] + [\lambda]^T[K_1 - K_o] \cdot \frac{\sqrt{\alpha}}{4} \qquad 2.40$$

will indicate the location and the value of maximum coefficient of distribution of longitudinal moment. Since a longitudinal beam may not lie under this reference station, this coefficient ($K_{\alpha max}$) is interpolated for the nearest

beam centreline, assuming a linear variation. Thus

$$M_{xmax} = 1.1M_{xmean} K_{\alpha max} \qquad \ldots 2.41$$

where M_{xmean} is average moment per beam of the bridge, equal to the applied moment divided by number of beams. M_{xmax} is the bending moment for which each longitudinal beam will be designed.

2.8 Coefficient for Distribution of Transverse Moments

The transverse moment M_y is given by equation (2.12). If w as given by equation (2.35) is put in equation (2.12) we get M_y in the form

$$M_y = \sum_{m=1}^{\infty} \mu_{m\theta} \, bHm \sin \frac{m\pi x}{L} \qquad \ldots 2.42$$

where $\mu_{m\theta}$ is a coefficient dependent on θ, α, e/b and y/b, and Hm is the amplitude of the load for various terms in Fourier's series. In the two limiting cases μ_o and μ_1 are defined for α = o and 1 respectively in the same way as K_o and K_1 were defined. We have thus [16]

$$\begin{aligned} \mu_o = \frac{1}{\sqrt{2}\,\pi\theta(\sinh^2 2\lambda b - \sin^2 2\lambda b)} \Big(& 2\sinh\lambda(b+y)\,\sin\lambda(b+y) \cdot \\ & [\sinh 2\lambda b \cos\lambda(b+e) \cosh\lambda(b-e) - \sin 2\lambda b \cosh\lambda(b+e) \cdot \\ & \cos\lambda(b-e)] + [\cosh\lambda(b+y)\sin\lambda(b+y) - \sinh\lambda(b+y)\cos\lambda(b+y)] \\ & \{\sinh 2\lambda b[\sin\lambda(b+e)\cosh\lambda(b-e) - \cos\lambda(b+e)\sinh\lambda(b-e)] \\ & + \sin 2\lambda b[\sinh\lambda(b+e)\cos\lambda(b-e) - \cosh\lambda(b+e)\sin\lambda(b-e)]\}\Big) \end{aligned} \qquad \ldots 2.43$$

and

$$\mu_1 = -\frac{1}{4\sigma\sinh^2\sigma}\cdot$$

$$\Bigg(\left\{\frac{[(1-\nu)\sigma\cosh\sigma-(1+\nu)\sinh\sigma]\cosh\theta\psi-\theta\psi(1-\nu)\sinh\sigma\,\sinh\theta\psi}{(3+\nu)\sinh\sigma\,\cosh\sigma-(1-\nu)\sigma}\psi.\right\}$$

$$\left\{[(1-\nu)\sigma\cosh\sigma-(3+\nu)\sinh\sigma]\cosh\theta\varphi-(1-\nu)\sinh\sigma\cdot\theta\varphi\cdot\sinh\theta\varphi\right\}$$

$$+\left\{\frac{[(1-\nu)\sigma\cosh\sigma+2\sinh\sigma]\sinh\theta\psi-(1-\nu)\sinh\sigma\cdot\theta\psi\cosh\theta\psi}{(3+\nu)\sinh\sigma\,\cosh\sigma+(1-\nu)\sigma}\psi.\right\}$$

$$\left\{(1-\nu)\sigma\cosh\sigma\,\sinh\theta\varphi-(1-\nu)\sinh\sigma\cdot\theta\varphi\cdot\cosh\theta\varphi\right\}$$

$$+[(1-\nu)\sigma\cosh\sigma-(1+\nu)\sinh\sigma]\cosh\theta\chi-(1-\nu)\sinh\sigma\cdot\theta\chi\sinh\theta\chi\Bigg) \qquad \ldots 2.44$$

where ν is poisson's ratio.

Again for a practical case α lies between zero and one. So μ_α is interpolated according to the formula

$$\mu_\alpha = \mu_o + (\mu_1 - \mu_o)\sqrt{\alpha} \qquad \ldots 2.45$$

2.9 Maximum Transverse Moments

The wheel position for 70R load to cause maximum transverse bending moment at the midspan is shown in Fig. 2.7. Rowe has shown [3] that the maximum transverse moment occurs at $x = \frac{L}{2}$ and $y = o$. In the present work transverse beams are provided at supports, quarter points and midspan. All transverse beams are designed and detailed identical to that at midspan. Due to these reasons, Rowe gives graphs for μ_o and μ_1 at reference station o.

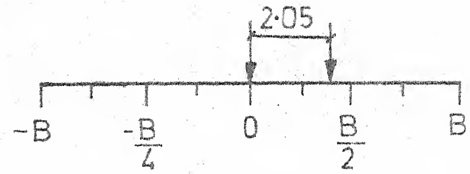
Coming back to the equation (2.42), $\mu_{m\theta}$ is the value of μ_α as given by 2.45 for flexural parameter mθ. However, since only the centre of the bridge need be considered, the appropriate values of x in the equation (2.42) is $x = \frac{L}{2}$, hence $\sin \frac{m\pi x}{L}$ is alternately plus and minus one for the odd terms of the series and zero for even terms of the series. The value of $\mu_{m\theta}$ in this equation can be determined from the influence curves for μ_o and μ_1 as given by equations (2.43) and (2.44) respectively for flexural parameter θ, 3θ, 5θ etc. $(\Sigma \mu_o)_{m\theta}$ and $(\Sigma \mu_1)_{m\theta}$ is the summation of the ordinates in the influence curves for μ_o and μ_1 respectively, the ordinates being measured under the specified position of wheels in the transverse section.

Hm is the amplitude of m^{th} term in the Fourier series for the load. The amplitude of the Fourier series for 70R loading as given in [4] is

$$Hm = \frac{2P}{L}\left\{\sin \frac{m\pi u_1}{L} + \sin \frac{m\pi u_2}{L} + \dots + \sin \frac{m\pi u_7}{L}\right\} \qquad \dots 2.54$$

where u's are distances of the respective axles from one support. Thus the maximum transverse moment can be written as

$$M_{ymax} = \frac{2b}{L}\left\{\Sigma \mu_\theta \left(\sum_{n=1}^{7} P_n \sin \frac{\pi u_n}{L}\right) - \Sigma \mu_{3\theta}\left(\sum_{n=1}^{7} P_n \sin \frac{3\pi u_n}{L}\right) + \Sigma \mu_{5\theta}\left(\sum_{n=1}^{7} P_n \sin \frac{5\pi u_n}{L}\right)\right\} \qquad \dots 2.55$$



WHEEL POSITION OF CLASS AA LOAD
FOR CASE I

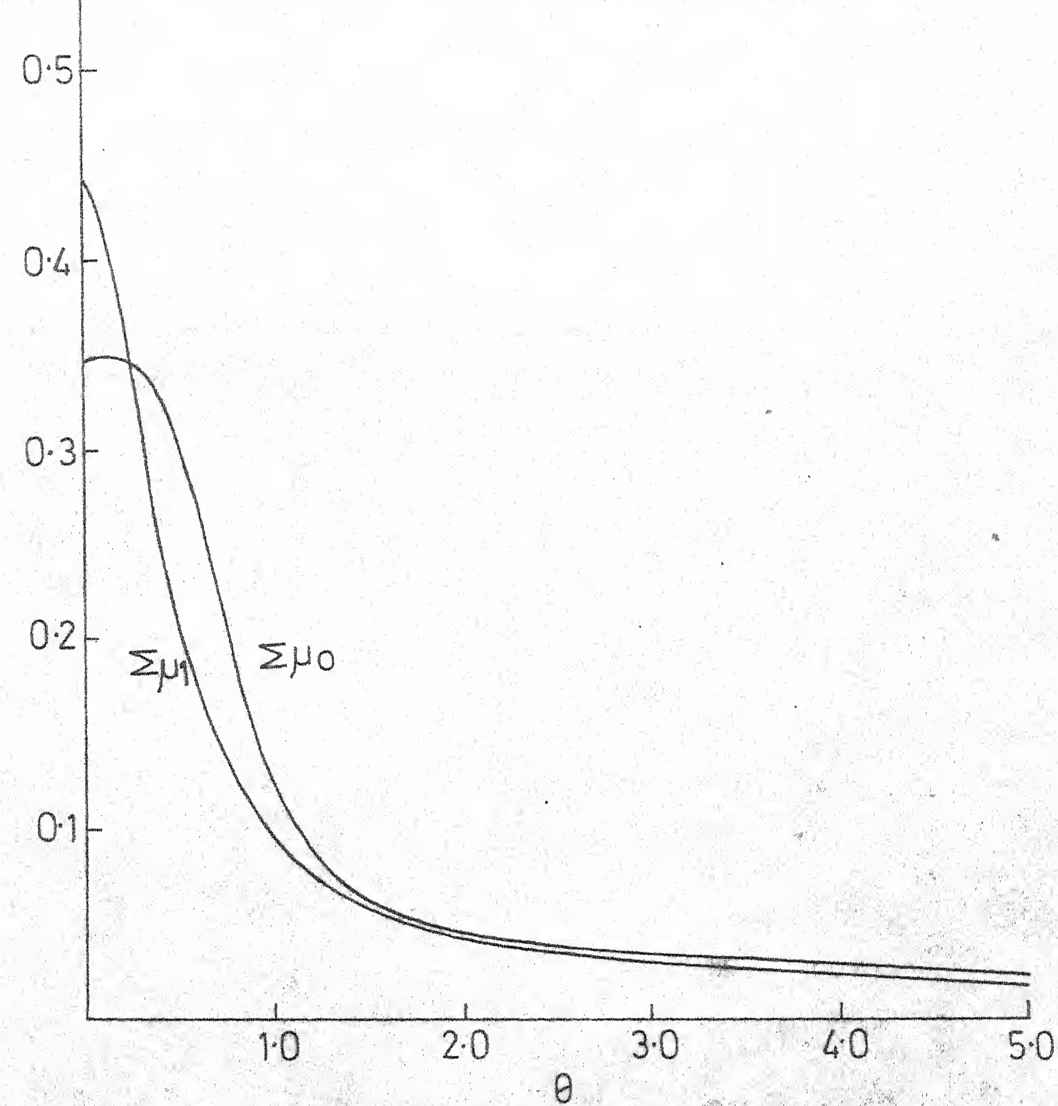


FIG 2·8 $\Sigma\mu_0, \Sigma\mu_1$, AT STATION O FOR CLASS AA LOAD CASE I

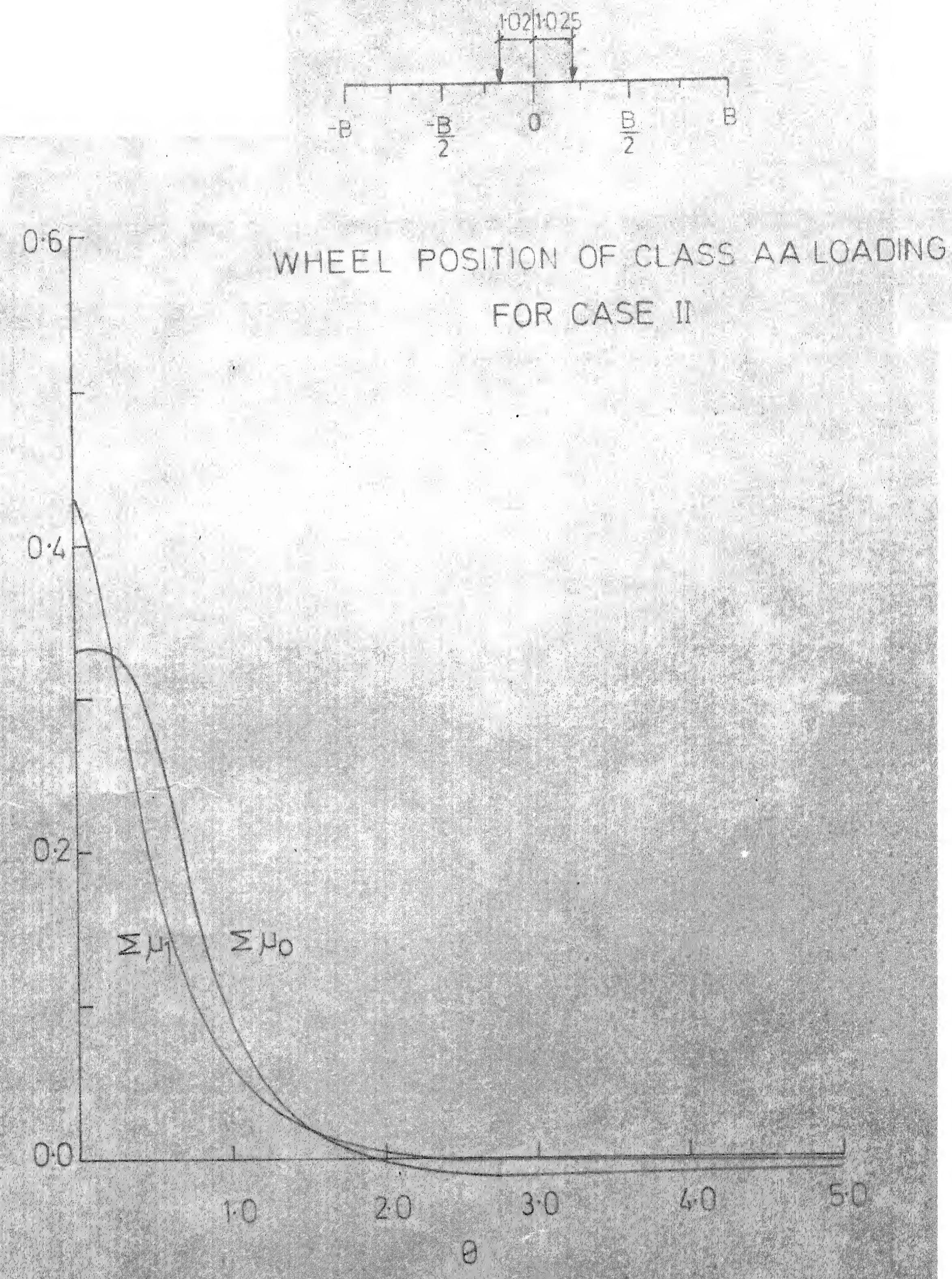


FIG 2·9 $\Sigma\mu_0, \Sigma\mu_1$ AT STATION 0 FOR CLASS AA L LOAD CASE II

FIG. 2.10 $\Sigma\mu_0, \Sigma\mu_1$ AT STATION 0 FOR 70R LOAD CASE I

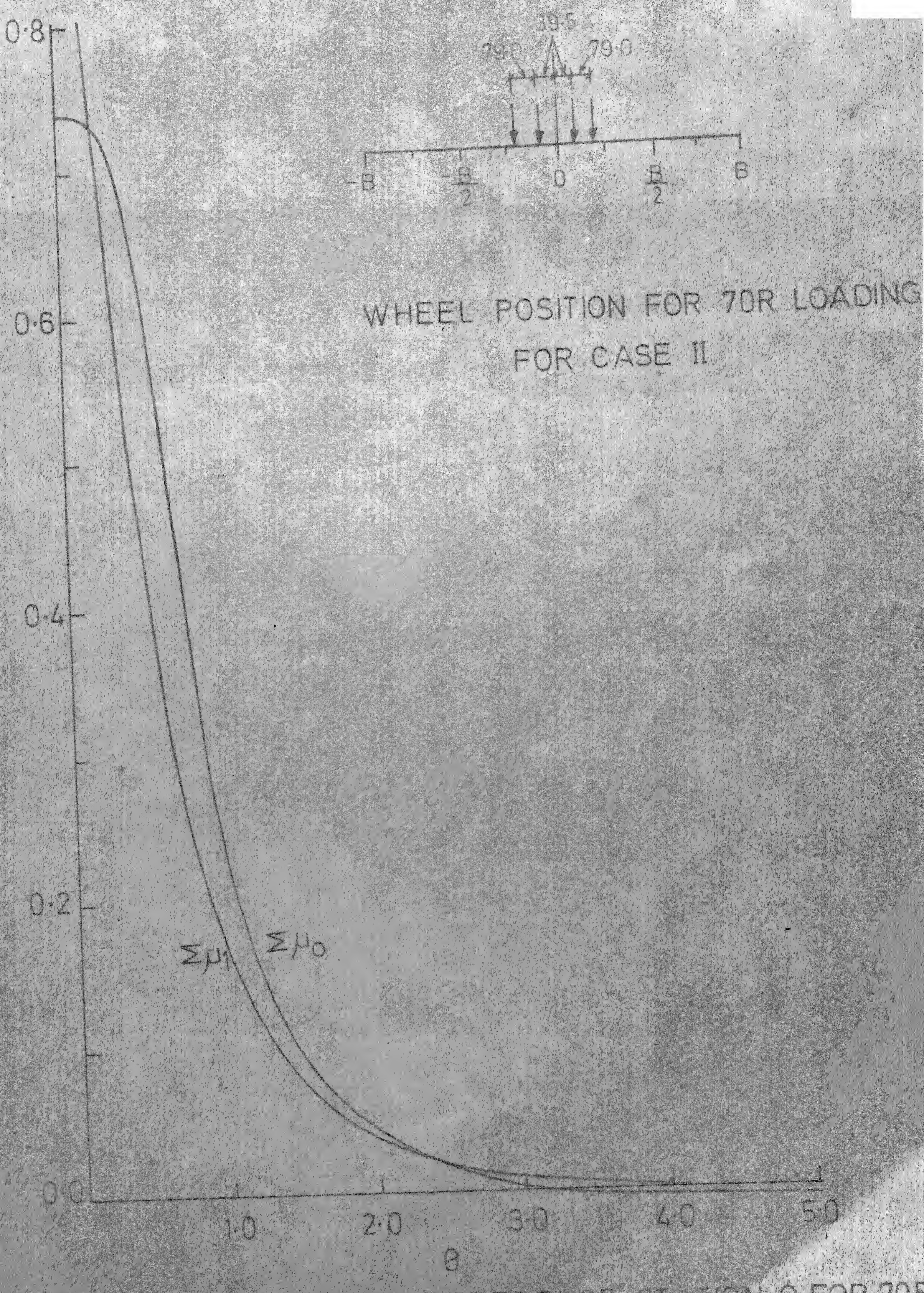


FIG. 2.11 $\Sigma\mu_0$ AND $\Sigma\mu_1$ AT REFERENCE STATION 0 FOR 70R LOAD CASE II

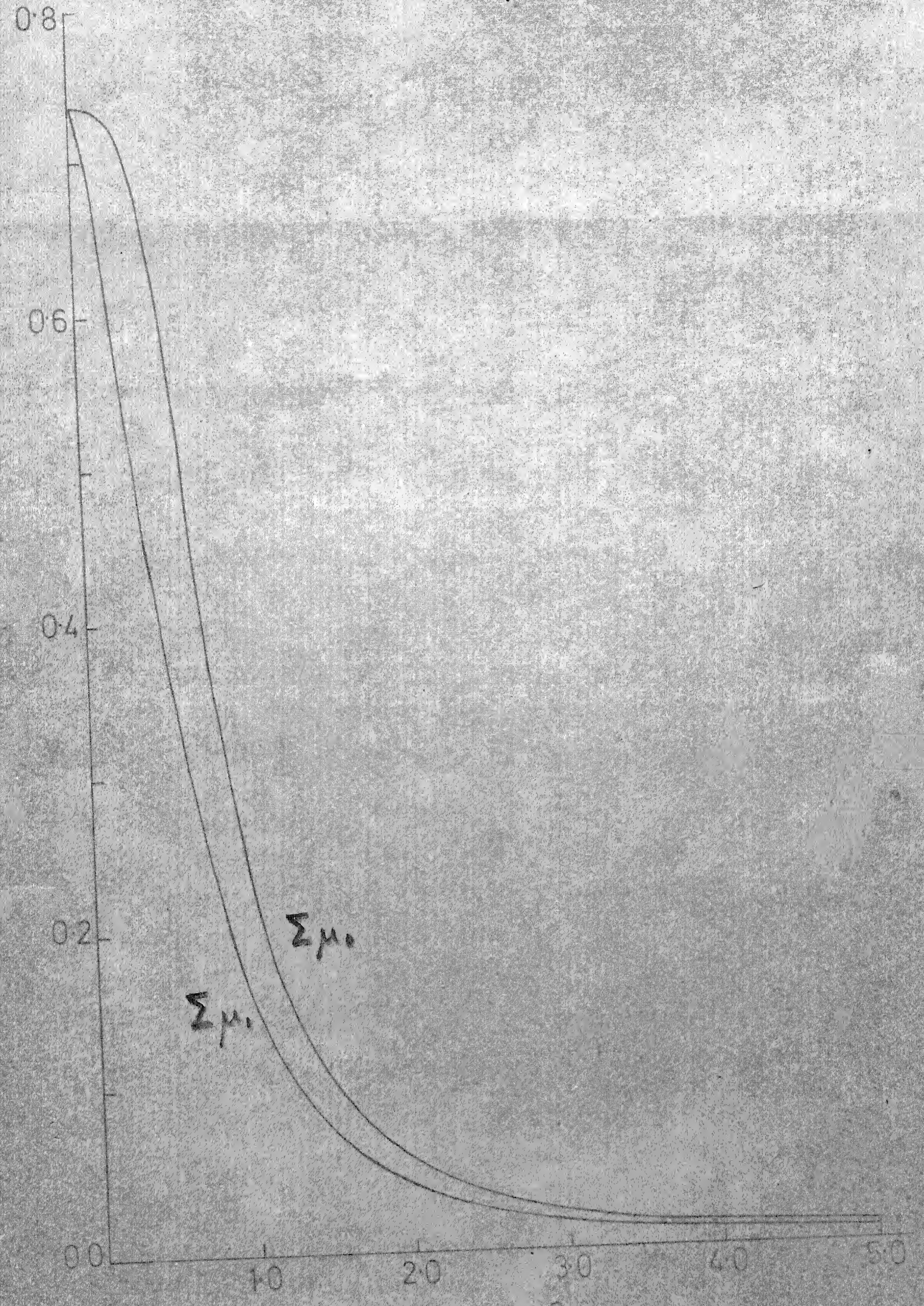


FIG 2·12 $\Sigma\mu_0$ AND $\Sigma\mu_1$ AT REFERENCE STATION 0 FOR 70R LOAD CASE I WITH γ=0 IN EQUATION 2·44

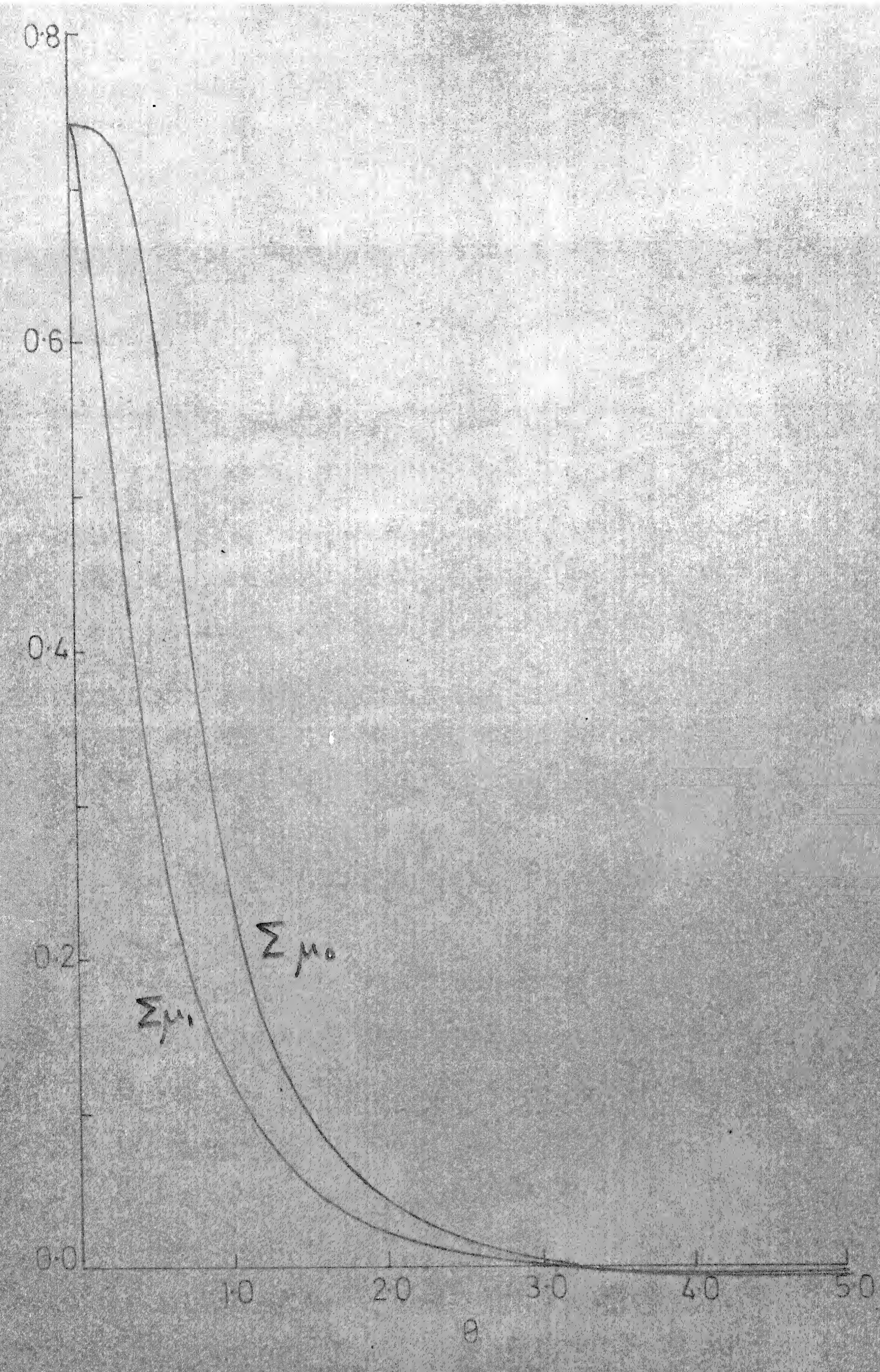


FIG. 2.13 $\Sigma\mu_0$ AND $\Sigma\mu_1$ AT REFERENCE STATION 0 FOR 70R LOADS CASE II WITH $\nu=0$ IN EQUATION 2.44

where $\Sigma \mu_{n\theta} = (\Sigma\mu_o)_{n\theta} + [(\Sigma\mu_1)_{n\theta} - (\Sigma\mu_o)_{n\theta}]\sqrt{\alpha}$ ...2.56

M_{ymax} is the design moment for transverse beams.

2.10 Graph for $\Sigma \mu_o$ and $\Sigma \mu_1$

It is obvious from equation (2.55) and (2.56) that computing the design transverse moment, M_{ymax}, is a tedious job. The determination of $\Sigma\mu_o$ and $\Sigma\mu_1$ involves drawing six influence curves, three each for μ_o and μ_1, for θ, 3θ and 5θ from graphs given by Rowe [3]. The ordinates at specified load position is marked on curves and summed up for a value of θ to get $\Sigma\mu_o$ and $\Sigma\mu_1$. Here an attempt has been made to draw the curve of $\Sigma\mu_o$ and $\Sigma\mu_1$ versus θ for 70R and class AA loading. The different graphs are as follows.

(i) Figure 2.8 is for Tracked class AA load travelling with one train of load coinciding with longitudinal centre line of the bridge. θ is from 0 to 5.0.

(ii) Figure 2.9 is for Tracked class AA load travelling the bridge in such a way that the two train of loads are symmetrically placed about the longitudinal centre-line. After $\theta > 2.25$, $\Sigma\mu_o$ and $\Sigma\mu_1$ becomes negative.

(iii) Fig. 2.10 is for 70R wheeled loads when one of the inner train of axle loads coincides with the longitudinal centre-line of the bridge.

(iv) Fig. 2.11 is for 70R wheeled loads when the train of axle loads are symmetrically placed about the longitudinal centre-line of bridge. Here also after $\theta > 4.0$, both $\sum \mu_0$ and $\sum \mu_1$ are negative.

(v) Fig. 2.12 is same as Fig. 2.10. The difference is that in equation (2.44) Poisson's ratio has been made zero.

(vi) Fig. 2.13 is same as Fig. 2.11 except that $\nu = 0$ in this case.

IBM 1800 was used to generate 250 points on these curves. The curves were drawn using IBM 1627 plotter.

CHAPTER III

DESIGN OF BRDIGE DECK

3.1 Introduction

The working load method has been used in designing the individual beams forming the deck. In fully prestressed beams cracks are not allowed to be developed in the section and material thus behaves as homogeneous and linearly elastic. Since the I.R.C. recommendation [17] does not allow any tensile stress to be developed in the beam, the use of working load method for designing the beams is justified.

The next section begins with a brief discussion on working load design. This is followed by statement of the problems taken up in the present work and the formulation of the optimum design problem. Chapter ends up with discussion on algorithms used for unconstrained minimization and linear search.

3.2 Working Load Method of Design of Beams

Let F_i be the prestressing force required at an eccentricity e for a prestressed concrete beam of cross-sectional area A. The section modulus for top fibre of beam be Z_t, the section modulus for bottom fibre of the beam be Z_b and applied live load bending moment be M_a while M_d is dead load bending moment.

The stress in bottom fibre at transfer condition is given by

$$\frac{F_i e}{Z_b} + \frac{F_i}{A} - \frac{M_d}{Z_b} \leq f_{bi} \qquad \ldots 3.1$$

where f_{bi} is allowable stress in bottom fibre at transfer. The stress in top fibre at transfer condition is given by

$$-\frac{F_i e}{Z_t} + \frac{F_i}{A} + \frac{M_d}{Z_t} \geqslant f_{ti} \qquad ...3.2$$

where f_{ti} is allowable stress in top fibre at transfer. Similarly the top fibre stress and bottom fibre stress at working condition are given by

$$-\frac{\eta F_i e}{Z_t} + \frac{\eta F_i}{A} + \frac{M_d}{Z_t} + \frac{M_a}{Z_t} \leqslant f_t \qquad ...3.3$$

and

$$\frac{\eta F_i e}{Z_b} + \frac{\eta F_i}{A} - \frac{M_d}{Z_b} - \frac{M_a}{Z_b} \geqslant f_b \qquad ...3.4$$

respectively. Where f_t is allowable stress in top fibre, f_b is allowable stress in bottom fibre at working conditions and η is loss in prestressing force.

For a section of known concrete grade and sectional properties only F_i and e are unknowns in the equations (3.1) to (3.4). But before F_i and e are found, expressions for minimum section modulus required at top and bottom fibres in a beam will be developed.

By multiplying equation (3.1) by η and substracting equation (3.4) from it

$$(1-\eta)\frac{M_d}{Z_b} + \frac{M_a}{Z_b} \leqslant \eta f_{bi} - f_b \qquad ...3.5$$

if equation (3.5) is taken as an equality the minimum section modulus required at bottom fibre Z_{bmin} is given by

$$Z_{bmin} = \frac{M_a + (1-\eta)M_d}{\eta f_{bi} - f_b} \qquad ...3.6$$

Similarly from equation (3.2) and (3.3) the minimum section modulus required at top Z_{tmin} is given by

$$Z_{tmin} = \frac{M_a + (1-\eta)M_d}{f_t - \eta f_{ti}} \qquad \ldots 3.7$$

The values of minimum eccentricity and corresponding prestressing force is worked out by satisfying equations (3.1) and (3.3), which gives

$$F_i = \frac{f_t Z_t + \eta f_{bi} \cdot Z_b - M_a - (1-\eta)M_d}{(Z_b + Z_t)\eta} \cdot A \qquad \ldots 3.8$$

$$e_{min} = \frac{Z_t}{A} + \frac{Z_b + Z_t}{A} \left(\frac{M_a + M_d - f_t Z_t}{f_t Z_t + \eta f_{bi} Z_b - M_a - (1-\eta)M_d} \right) \qquad \ldots 3.9$$

Similarly the maximum eccentricity and the corresponding prestressing force is worked out by satisfying equations (3.2) and (3.4), which gives

$$e_{max} = \frac{Z_t}{A} + \frac{Z_b + Z_t}{A} \left(\frac{M_d - f_{ti} Z_t}{M_a + (1-\eta)M_d + f_b Z_b + \eta f_{ti} Z_t} \right)\eta \qquad \ldots 3.10$$

$$\text{and } F_i = \frac{M_a + (1-\eta)M_d + f_b Z_b + \eta f_{ti} Z_t}{\eta(Z_t + Z_b)} A \qquad \ldots 3.11$$

The actual eccentricity provided is generally one which gives a maximum of 0.1h as cover. Where h is the overall depth of the beam. This eccentricity is accepted if it lies between e_{min} and e_{max} as found by equations (3.9) and (3.10) respectively. So once the actual eccentricity e is known the prestressing force can be found by taking equation (3.4)

as an equality. Thus

$$F_i = \frac{M_a + M_d + f_b Z_b}{\eta(e + \frac{Z_b}{A})} \qquad \ldots 3.12$$

The cables are laid for F_i in a parabolic profile.

With F_i, e and other variables defining A, Z_t, Z_b known checks are made to find the feasibility of the solution. The checks are

(i) The section modulus provided for top fibre Z_t has to be more than Z_{tmin} given by equation (3.7).

(ii) The section modulus provided for bottom fibre Z_b has to be more than Z_{bmin} given by equation (3.6).

(iii) The four stress inequalities given by equations (3.1) to (3.4) have to be satisfied.

(iv) The eccentricity provided has to be between the values obtained by equations (3.9) and (3.10).

(v) The section has to be under-reinforced as given by IS-1343-1965 [19].

(vi) The ultimate applied bending moment given by $2.5M_a + 1.5M_d$ has to be less than ultimate capacity of section.

3.3 Transverse Beam Design

The dead load on the bridge deck can be taken as uniformly distributed on the entire span. Due to this symmetric load all the longitudinal beams will deflect by an equal amount and the transverse beams do not get loaded.

Thus only moment due to live load on deck will cause stresses to develop in the transverse beam. The transverse beams are designed as a member which is in uniform stress due to prestressing force. This is possible if the eccentricity is zero. The expression for the four stress inequalities become

$$\frac{F_t}{A_t} \leq f_{bi} \qquad \ldots 3.13$$

$$\frac{F_t}{A_t} \geq f_{ti} \qquad \ldots 3.14$$

$$\frac{\eta F_t}{A_t} + \frac{M_y}{Z_{tt}} \leq f_t \qquad \ldots 3.15$$

$$\frac{\eta F_t}{A_t} - \frac{M_y}{Z_{bt}} \geq f_b \qquad \ldots 3.16$$

where F_t is transverse prestressing force, A_t is area of transverse beam, Z_{tt} and Z_{bt} are section modulus for top and bottom fibre of transverse beam. The actual prestressing is given by

$$F_t = \frac{M_y \cdot A_t}{\eta Z_{bt}} \qquad \ldots 3.17$$

The stress in bottom fibre at transfer condition as given by equation (3.13) and stress in top fibre at working condition as given by equation (3.15) is checked, to ascertain if they are within permissible limits. The cables corresponding to F_t is laid straight. This is done so that

it does not cross the cables of longitudinal beams.

3.4 Design for Under-Reinforced Section

To prevent failure of prestressed beams by crushing of concrete the clause 6.8.2.2(c) of IS-1363 [19] is effective. This is achieved by putting an upper limit to the steel to be used in the section given by

$$\frac{A_{tsw} \cdot Fs}{b'd\ Fc} \ngtr 0.240 \qquad \ldots 3.18$$

where $A_{tsw} = A_{ts} - A_{tsf}$

A_{ts} = area of steel provided

A_{tsf} = 0.68 Fc (b-b') t/Fs

b = width of the flange

b' = width of the web

d = depth from the compression edge to the centre of the prestressing steel in the tensile zone.

t = average thickness of flange of a flanged member

For such beams the ultimate moment of resistance (M_{us}) is calculated as follows [19]

(i) if t d/4

$$M_{us} = K_{us} \cdot A_{ts} \cdot Fs \cdot d[1 - 0.75\ \frac{A_{ts} \cdot Fs}{b \cdot d \cdot Fc}] \qquad \ldots 3.19$$

(ii) if t d/4

$$M_{us} = K_{us} \cdot A_{tsw} \cdot Fs \cdot d[1 - 0.75\ \frac{A_{tsw} \cdot F_s}{b' \cdot d \cdot Fc}]$$

$$+ 0.7Fc \cdot t(b-b')(d-0.5t) \qquad \ldots 3.20$$

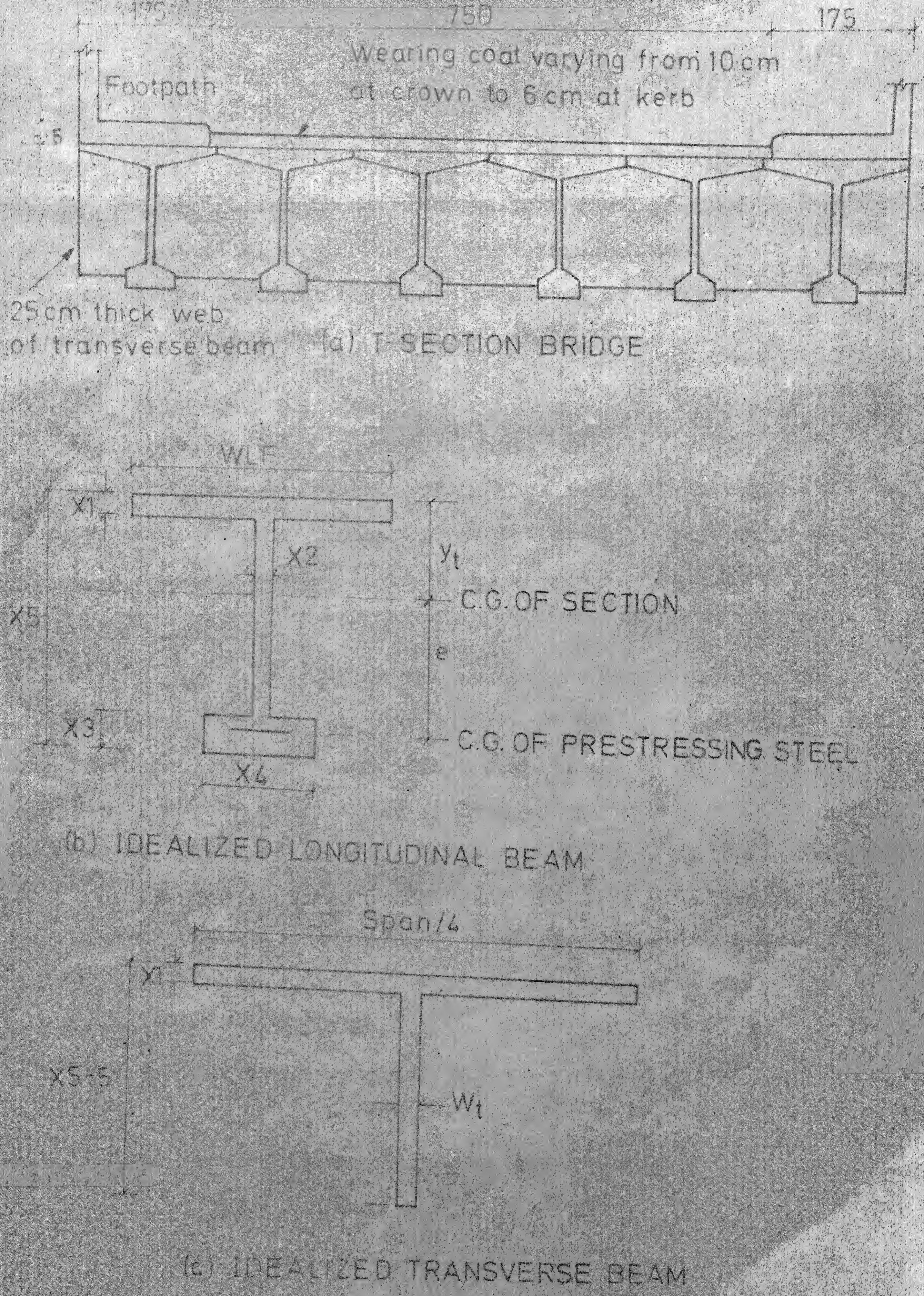


FIG 3.1 T-SECTION BRIDGE DECK

K_{us} = 1.0 if tensioned steel is effectively bonded to concrete, and equal to 0.7, when steel is unbonded.

Fs is ultimate tensile strength of prestressing steel,

Fc is cube strength of concrete at 28 days.

3.5 Optimum Design Problem

3.5.1 Design Variables: As already mentioned earlier two types of cross-sections used in the longitudinal beams of bridge decks have been selected for study in the present work, they are (i) T-section and (ii) Hollow Box section.

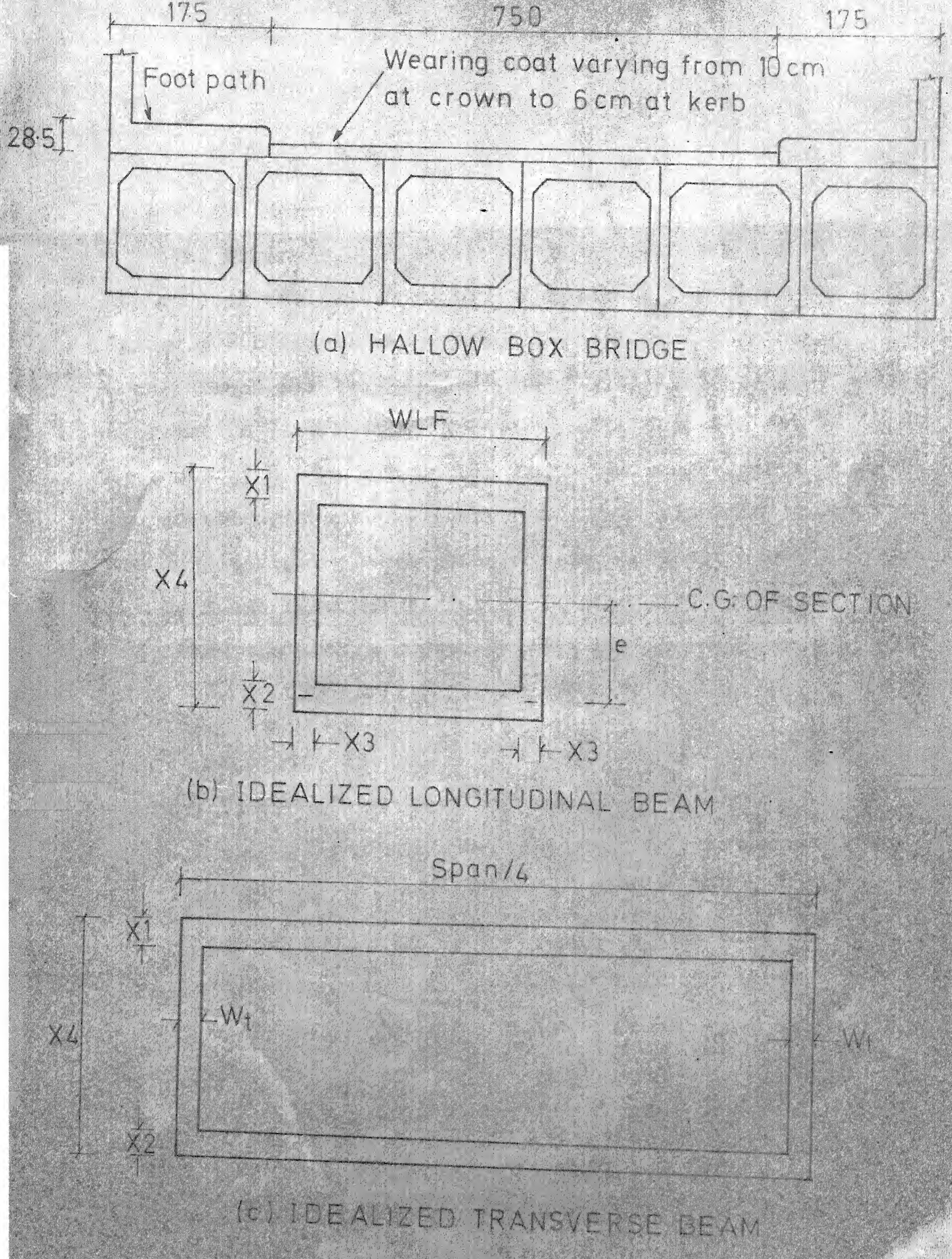
(i) T-section

Figure 3.1(a) shows the transverse cross-section of a bridge of 2-lane with foot path on both sides. If number of beams used in the longitudinal direction is known (say N) and 2b is the total width of bridge, the width of top flange becomes automatically fixed given by $WLF = \frac{2b}{N}$ ...3.21

The Figure 3.1(b) represents the idealized T-section used in the deck, the dimension which completely define the geometry of the section are X1 to X5. The transverse beam is shown in Figure 3.1(c). The prestressing force in longitudinal direction and transverse directions and their eccentricities can be found from these dimensions.

The design vector for T-section bridge, therefore, is

$$[X]^T = [X1, X2, X3, X4, X5] \qquad ...3.22$$



(a) HALLOW BOX BRIDGE

(b) IDEALIZED LONGITUDINAL BEAM

(c) IDEALIZED TRANSVERSE BEAM

FIG. 3·2 HOLLOW BOX SECTION BRIDGE DECK

(ii) Hollow Box section

The Figure 3.2(a) shows the transverse section of a bridge deck made up of number of Hollow Box section beams, Figure 3.2(b) shows the idealized section of beams in longitudinal direction and Figure 3.3(c) is idealized section of beams in transverse direction. The variable are X1 to X4 as shown. Top and bottom width of flanges are fixed with the number of beams predetermined. Here also the transverse and longitudinal prestressing forces and their eccentricities can be found once the variables X1 to X4 are assigned.

Thus the design vector in this problem is

$$[X]^T = [X1, X2, X3, X4] \qquad \ldots 3.23$$

3.5.2 Objective Function: The objective function (OBJ) for both problems are the cost of deck given by

OBJ = Cost of concrete in longitudinal and transverse beams (C1)
+ Cost of prestressing cables with its sheathing and anchorage in longitudinal beams (C2) + Cost of prestressing cables with its sheathing and anchorage in transverse beam (C3) ...3.24

where for T-section shown in Figure 3.1

$$C1 = \left\{ A \cdot L + 5 \cdot W_t [WLF(X5-5) - (A - 5 \cdot X4)] \right\} CC \cdot N \qquad \ldots 3.25$$

and A = area of concrete in longitudinal beam

W_t = thickness of transverse web

CC = cost of concrete/cm^3

$$C2 = [(CN - 1)\cdot CHTW + ADC + CTW](1 + \frac{8e^2}{3L^2})L\cdot N + CAC\cdot CN\cdot N \qquad \ldots 3.26$$

in this CN is total number of sheathing required for cables.

CHTW cost of cable consisting of 12 numbers of 7m.m dia wires with sheathing per cm.

ADC cost of wires of 7m.m dia required in the last cable/cm.

CAC cost of anchorage for cables of 12 numbers of 7m.m dia wires per cable.

CTW cost of one sheathing per cm

and $$C3 = TCN\cdot 2b\cdot CHTW + TCN\cdot CAC \qquad \ldots 3.27$$

TCN = total number of cables required in transverse direction.

For a Hollow Box section shown in Fig. 3.2, the different costs are

$$C1 = [A\cdot L + 5.0(X4 - X2 - X1)\cdot(WLF - 2\cdot X3)\cdot W_t]CC\cdot N \qquad \ldots 3.28$$

$$C2 = [(CN-1)\cdot CHTW + ADC\cdot CTW][1 + \frac{8e^2}{3L^2}]L\cdot N + CAC\cdot CN\cdot N \qquad \ldots 3.29$$

and $$C3 = TCN\cdot 2b\cdot CHTW + TCN\cdot CAC \qquad \ldots 3.30$$

3.5.3 Constraints: The constraints are described here in the normalized form.

3.5.3.1 Constraints Common to the Two Problems

(i) Minimum section modulus constraints

(a) For top fibre

$$g_1 = \frac{Z_{tmin}}{Z_t} - 1.0 \leq 0.0 \qquad \ldots 3.31$$

Z_{tmin} is given by equation (3.7), Z_t is actual section modulus at top fibre.

(b) For bottom fibre

$$g_2 = \frac{Z_{bmin}}{Z_b} - 1.0 \leq 0.0 \qquad \ldots 3.32$$

Z_{bmin} is given by equation (3.6), Z_b is actual section modulus at bottom fibre.

(ii) Permissible stress constraints for longitudinal beams.

(a) Denoting actual bottom fibre stress at transfer condition as f_{bai}.

$$g_3 = \frac{f_{bai}}{f_{bi}} - 1.0 \leq 0.0 \qquad \ldots 3.33$$

where f_{bi} is allowable stress in bottom fibre at transfer condition.

(b) Denoting actual top fibre stress at transfer condition as f_{tai}

$$g_4 = \frac{f_{ti}}{f_{tai}} - 1.0 \leq 0.0 \qquad \ldots 3.34$$

where f_{ti} is allowable stress in top fibre at transfer condition.

(c) Denoting actual bottom fibre stress at working condition as f_{ba}

$$g_5 = \frac{f_b}{f_{ba}} - 1.0 \leq 0.0 \qquad \ldots 3.35$$

where f_{ba} is allowable stress in bottom fibre at working condition.

(d) Denoting actual top fibre stress at working condition as f_{ta},

$$g_6 = \frac{f_{ta}}{f_t} - 1.0 \leq 0.0 \qquad ...3.36$$

where f_t is allowable stress in top fibre at working condition.

(iii) Ultimate strength constraint

$$g_7 = \frac{M_{ult}}{M_{us}} - 1.0 \leq 0.0 \qquad ...3.37$$

where $M_{ult} = 2.5M_a + 1.5M_d$ and M_{us} is given by equation (3.20).

(iv) Permissible stress constraints for transverse beams

(a) Denoting the actual stress at transfer condition in bottom fibre of transverse beam as f_{btai}

$$g_8 = \frac{f_{btai}}{f_{bi}} - 1.0 \leq 0.0 \qquad ...3.38$$

(b) Denoting the actual stress at working condition in top fibre of transverse beam as f_{tta}

$$g_9 = \frac{f_{tta}}{f_t} - 1.0 \leq 0.0 \qquad ...3.39$$

3.5.3.2 Additional Constraints Imposed on T-section

(i) For under reinforced design

$$g_{10} = \frac{A_{tsw}\ Fs}{0.24\ b'\ d\ Fc} - 1.0 \leq 0.0 \qquad ...3.40$$

terms appearing in equation (3.40) are defined in section 3.4. In case of few T-sections A_{tsw} given by equation (3.18a) becomes negative. As such sections are very much under reinforced, this constraint is

redefined by assigning it a value equal to 0.0.

(ii) Constraint on depth of the section

$$g_{11} = \frac{15 \cdot X5}{L} - 1.0 \leq 0.0 \qquad \ldots 3.41$$

where L is span of bridge and X5 is overall depth of T-section. This constraint restricts the depth to L/15.

(iii) Heavy top design of T-section

$$g_{12} = \frac{Z_b}{Z_t} - 1.0 \leq 0.0 \qquad \ldots 3.41a$$

This constraint will give section modulus in top fibre more than bottom fibre.

(iv) Geometric constraints

$$g_{13} = \frac{13.0}{X1} - 1.0 \leq 0.0 \qquad \ldots 3.42$$

$$g_{14} = \frac{14.0}{X2} - 1.0 \leq 0.0 \qquad \ldots 3.43$$

$$g_{15} = \frac{20.0}{X3} - 1.0 \leq 0.0 \qquad \ldots 3.44$$

These constraints mean that the thickness of top flange is to be more than 13.0cm, the thickness of web is to be more than 14.0cm and thickness of bottom flange is to be more than 20.0cm.

Thus there are a total number of fifteen constraints in T-section.

3.5.3.3 Additional Constraints Imposed on Hollow Box Section

(i) For under-reinforced design

$$g_{10} = \frac{A_{tsw}\ Fs}{0.24\ b'\ d\ Fs} - 1.0 \leq 0.0 \qquad \ldots 3.45$$

The terms coming above are defined in section 3.4.

(ii) Constraint on overall depth

$$g_{11} = \frac{15 \cdot X4}{L} - 1.0 \leq 0.0 \qquad \ldots 3.46$$

This constraint restricts the depth to L/15.

(iii) Geometric Constraints

$$g_{12} = \frac{14.0}{X1} - 1.0 \leq 0.0 \qquad \ldots 3.47$$

$$g_{13} = \frac{14.0}{X2} - 1.0 \leq 0.0 \qquad \ldots 3.48$$

$$g_{14} = \frac{14.0}{X3} - 1.0 \leq 0.0 \qquad \ldots 3.49$$

The thicknesses of top flange, bottom flange and web is to be more than 14 cms. Thus there are a total of fourteen constraints in the Hollow Box section.

3.5.4 Formulation of Optimum Design Problem

Thus the optimum design problems for T-section and Hollow Box section bridges turns out as:

(i) T-Beam Section

Minimize OBJ(X1, X2, X3, X4, X5)

subject to

$$g_j \leq 0.0 \;; \quad j = 1, 2, \ldots, 15$$

(ii) Hollow Box section

Minimize OBJ(X1, X2, X3, X4)

subject to

$$g_j \leq 0.0 \;; \quad j = 1, 2, \ldots, 14$$

where objective function, OBJ and constraints g_j, for each case are defined in sections immediately precceding.

3.5.5 Parametric Study

Parametric study has been done to have an idea of change in cost. The parameter for both the problems is number of longitudinal beams. This number changes from 5 to 9 for both the sections. It is suggested [7] that the number of beam has to be more than 4 for the orthotropic plate analysis to be justified. That is why the minimum number of beams considered is 5.

3.6 Optimization Technique

The interior penalty function approach [5] is employed in solving the non-linear programming problem of finding the minimum of OBJ given by equation (3.24) subject to constraints g_j ; $j = 1, 2, \ldots, M$, given by equations (3.31) to (3.49). M is the total number of constraints. As there is no equality constraint to be handled the penalty function is defined as

$$\varphi\,(\bar{X},\ r) = OBJ - r \sum_{j=1}^{M} \frac{1}{g_j} \qquad \ldots 3.50$$

where r is penalty parameter.

Sequential unconstrained minimization technique is used to find the minimum of $\varphi(\bar{X}, r)$. This essentially means finding minimum of $\varphi(\bar{X}, r)$ as r sequentially goes to zero, and in such a situation minimum of $\varphi(\bar{X}, r)$ is minimum of OBJ.

The unconstrained minimization of $\varphi(\bar{X}, r)$ has been performed by the algorithm due to Davidon Flecther Powell [22]. In this method the move $\bar{S}_i$ is generated as

$$\bar{S}_i = -[H]_i\ \bar{D}f_i \qquad \ldots 3.51$$

$\overline{Df}_i$ is gradient at point $\overline{X}_i$; [H] is a positive definite matrix of n x n size, where n is number of variable in the problem. To start with [H] is an identity matrix.

The new point $\overline{X}_{i+1}$ is obtained by

$$\overline{X}_{i+1} = \overline{X}_i + \beta_i \cdot \overline{S}_i \qquad \ldots 3.52$$

where β_i is a scalar greater than zero chosen to minimize φ function along $\overline{S}_i$, starting from $\overline{X}_i$. The matrix [H] is updated in each iteration as

$$[H]_{i+1} = [H]_i + \frac{(\beta_i \cdot \overline{S}_i)(\beta_i \cdot \overline{S}_i)^T}{(\beta_i \cdot \overline{S}_i)^T \cdot \overline{Y}_i} - \frac{[H]_i(\overline{Y}_i)(Y_i)^T[H]_i}{(Y_i)^T[H]_i(\overline{Y}_i)} \qquad \ldots 3.53$$

where $$\overline{Y}_i = \overline{Df}_{i+1} - \overline{Df}_i \qquad \ldots 3.54$$

The linear search performed in $\overline{S}_i$ to obtain β_i was done by Fibonacci method [21]. This makes use of the sequence of Fibonacci numbers for placing the experiments in the initial range of uncertaincy, and reduces the range by 0.00103 with help of fifteen experiments. The sequence of Fibonacci numbers are given as

$$F_o = F_1 = 1$$

and $$F_n = F_{n-1} + F_{n-2} \; ; \quad n = 2, 3, 4, \ldots \qquad \ldots 3.55$$

yielding the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

CHAPTER IV

RESULTS AND DISCUSSIONS

4.1 Introduction

The minimum cost design of two bridge decks each having different types of cross-sections of the longitudinal beams has been carried out in the present work. The illustrative examples of spans 24m and 30m in each case have been presented. As a result of the present work a computer program has been developed for minimum cost design of bridge decks. The listing of the program and the subroutines in Fortran IV is given in the appendix.

The computer sub-program referred to as TEEBRI performs the minimum cost design of bridge decks constisting of a number of T-section longitudinal beams, while the computer program referred to as BOXGIR is for minimum cost design of bridge decks consisting of a number of Hollow Box sections. The formulation of these problems is given in Chapter III.

Cost of different items required to find the cost of bridge deck is given in Table 4.1. The cost table has been taken from reference 9. The allowable stresses in concrete and steel wire is presented in Table 4.2. This table has been generated from reference 5.

Table 4.1 : Cost of Items

Item	Description	Rate
Concrete		
(a)	Grade of concrete M-350	265.00 Rs./m^3
(b)	Grade of concrete M-425	297.00 Rs./m^3
H.T.S. wires		
(a)	Cable consisting of 12 nos. of 7m.m dia wires of H.T.S. with sheathing	18.00 Rs./m
(b)	One untensioned wire of 7m.m dia	0.91 Rs./m
(c)	Two anchorage for cable consisting of 12 nos. of 7m.m dia wires	110.00 Rs./cable
(d)	Cost of sheathing for cables consisting of 12 nos. of 7m.m dia wires	7.00 Rs./m

Table 4.2 : Allowable Stresses

Item	Description of stress	Notation	Permissible Values
Concrete			
(a)	Allowable stress in bottom fibre at transfer	f_{bi}	
	(i) For concrete grade M-350		140.0 Kg/cm^2
	(ii) For concrete grade M-425		157.14 Kg/cm^2
(b)	Allowable stress in top fibre at transfer	f_{ti}	
	(i) For concrete grade M-350		0.0 Kg/cm^2
	(ii) For concrete grade M-425		0.0 Kg/cm^2
(c)	Allowable stress in top fibre at working	f_t	
	(i) For concrete grade M-350		110.0 Kg/cm^2
	(ii) For concrete grade M-425		122.86 Kg/cm^2
(d)	Allowable stress in bottom fibre at working	f_b	
	(i) For concrete grade M-350		0.0 Kg/cm^2
	(ii) For concrete grade M-425		0.0 Kg/cm^2
Steel	Ultimate strength of steel wire	F_s	15000 Kg/cm^2

4.2 Comparison of Results

The sections and spans taken up for study in present work also appear in the list of standard sections recommended by SERC handbook [9]. Furthermore, the sections corresponding to minimum cost of bridge decks have been recommended in the said handbook. The optimization technique used in finding these optimum sections in rather crude. In the referred work, the cost of bridge decks with beams increasing in depth by one centimeter is compared and the depth which corresponds to minimum cost is recommended as optimum design. This essentially is formulating the optimum design problem with one variable Moreover the cost of transverse prestressing cables has been neglected. As compared to this the computer programs developed in the present work handles the same optimization problem having five variables and fifteen constraints as formulated in Chapter III. In order to compare the results of the present work with that documented in reference 9 the sub-program TEEBRI was modified to solve the same problem but having five variables and thirteen constraints. The two constraints dropped are g_8 and g_9 which go to check the stresses in transverse beams as given by equations (3.38) and (3.39). The objective function was also modified to exclude the cost of transverse prestressing cables. The results given in Table 4.3 are for 24m span bridge deck of concrete grade M-425, the number of longitudinal beams considered are

Table 4.3 : Comparison of optimum design v/s standard section design of SERC (Span 24m, Concrete grade M-425)

Results from	Nos. of longitudinal beam	Variables at optimum					Area of concrete in longitudinal beam cm²	Prestressing force per Kg	Cost in Rs.	A
		X1	X2	X3	X4	X5				
Present work	6	13.55	14.35	31.22	50.00	138.65	5380.5	358966	55871	8.4
Reference 9	6	17.30	14.00	21.24	50.00	153.00	6158.0	328837	61000	
Present work	7	13.05	14.14	21.01	54.60	135.15	4609.8	313109	56850	9.15
Reference 9	7	16.00	14.00	21.24	50.00	153.00	5394.0	289210	62000	

A = Percentage reduction in cost as established in present work

six and seven. It can be concluded from Table 4.3 that the sections given in reference 9 do not quite correspond to minimum cost design. The present work has shown the reduction in cost by 8.4 percent in one case and 9.15 percent in other as is seen from Table 4.3.

4.3 Results of Parametric Study

The results of the parametric study obtained for the two illustrative examples as formulated in Chapter III can be categorized into the following.

(i) 30m span bridge decks comprising of T-section longitudinal beams, number of which changes from 5 to 9. (Table 4.4, serial number 1 to 5).

(ii) 24m span bridge decks comprising of T-section longitudinal beams number of which changes from 5 to 9. (Table 4.4 serial number 6 to 10).

(iii) 30m span bridge decks comprising of Hollow Box section longitudinal beams, number of which changes from 5 to 9. (Table 4.5, serial number 1 to 5).

(iv) 24m span bridge decks comprising of Hollow Box section longitudinal beams, number of which changes from 5 to 9 (Table 4.5, serial number 6 to 10).

In the above results the grade of concrete is M-350.

Table 4.4 : T-section Bridges

Span	Sl No	A_1	Variables at optimum					A_2	A_3	A_4	A_5	Cost in Rs.	Number of active constraints
			X1	X2	X3	X4	X5						
30m	1	5	16.32	14.36	23.55	76.56	194.20	7584.9	479787	12	1471507	85,840	2,3,5,11,14
	2	6	18.77	14.08	23.04	57.20	196.43	6910.2	404505	10	1533040	89,259	2,3,5,11,14
	3	7	18.00	14.11	22.18	57.90	190.60	6210.0	365546	9	1544551	92.546	2,3,5,11,14
	4	8	19.58	14.11	20.56	67.83	195.51	6253.0	332091	8	1560550	98,409	2,3,5,11,14,15
	5	9	20.60	15.14	20.35	49.72	198.06	5888.4	300880	8	1561781	106014	2,3,5,11,14,15
24m	6	5	13.93	15.65	32.85	67.27	152.01	6898.7	408554	10	1554249	64,834	2,3,5,11,13
	7	6	15.81	16.85	37.44	56.72	150.10	6551.5	358120	9	1517683	70,383	2,3,5,11
	8	7	13.91	14.07	33.53	38.85	154.50	4997.0	283304	7	1576800	64,601	2,3,5,11,13,14
	9	8	16.46	14.03	37.54	54.18	155.42	5699.8	277285	7	1595072	75,265	2,3,5,11,14
	10	9	18.50	20.32	39.90	41.43	152.26	5793.2	268984	7	1714943	83,876	2,3,5,11

A_1 = Nos. of longitudinal beams, A_2 = Area of concrete in logitudinal beams
A_3 = Prestressing force per longitudinal beam (Kg), A_4 = Nos. of cables in longitudinal beams.
A_5 = Total transverse prestressing force (Kg)

The clear cover to c.g. of steel is 0.1 times the depth of section

Table 4.5 : Hollow Box Section Bridges

span	Sl No	A_1	Variables at optimum				A_2	A_3	A_4	A_5	cost in Rs.	Number of active constraint
			X1	X2	X3	X4						
	1	5	14.15	14.15	14.11	176.9	10426.5	560110	14	1115193	97,987	5,12,13,14
	2	6	14.23	14.11	14.13	175.6	9313.5	498733	12	1122244	102405	5,12,13,14
30m	3	7	14.20	14.15	14.11	175.2	8554.8	451683	11	1126737	108794	5,12,13,14
	4	8	14.16	14.25	14.03	173.0	7918.2	418120	11	1130638	113404	5,12,13,14
	5	9	14.14	14.13	14.06	177.7	7608.4	384050	10	1136638	123740	5,12,13,14
	6	5	14.10	14.10	14.16	111.03	8529.0	561540	14	1367220	76846	5,6,12,13,14
	7	6	14.13	14.10	14.18	110.18	7477.9	492286	12	1398859	79612	5,6,12,13,14
24m	8	7	14.12	14.06	14.04	110.06	6698.6	435706	11	1400578	83874	5,6,12,13,14
	9	8	15.95	15.43	15.24	110.02	6680.1	413276	10	1426998	89693	5,12,13,14
	10	9	16.00	15.00	15.00	110.00	6125.0	375995	9	1448156	91273	5,12,13,14

A_1 = Nos. of longitudinal beams, A_2 = Area of concrete in longitudinal beam
A_3 = *Prestressing force per longitudinal beam*, A_4 = *Nos. of Cables per longitudinal beam*
A_5 = Total Transverse prestressing force

The clear cover to c.g. of steel is 0.1 times the depth of section

4.4 Discussions of Results

(a) T-Section Bridge:

In general it can be said that the cost of bridge increases with increase in number of longitudinal beams. Let us consider the 30m span bridge. The cost of 5 beam bridge is minimum. This cost is 0.926 times the cost of 7 beam bridge. The weight of one beam of 5 beam bridge deck is about 1.22 times the weight of one beam of 7 beam bridge deck. When the 5 beam bridge deck is compared with 9 beam bridge deck its cost is 0.8 times the cost of 9 beam bridge, and the weight is about 1.29 times the weight.

Taking the results of the 24m span bridge the cost of 5 beam bridge deck is about same as 7 beam bridge deck. But if weight of individual beams are compared the 5 beam bridge is about 1.38 times the weight of 7 beam bridge. Again comparing the 5 beam bridge with 9 beam bridge it is observed that 5 beam bridge is about 0.76 times the cost of 9 beam bridge and in weight they are 1.18 times heavier. The weight of individual beams can change overall economy at the time of launching of the bridge. A heavy beam will need more careful handling. Thus if in the span under consideration (24m and 30m) a 7 beam bridge deck is constructed there will be an average increase in the cost of bridge by about 4 percent, but the weight of individual beams to be handled will be about 23 percent less. Since the detailed data regarding

handling and launching of the beams was not available a rigorous study could not be pursued, however, it is felt that the 7 beam bridge deck will turn out to be over all economical.

The active nature of the constraint g_2 means that the optimum design point corresponds to a heavy top flange design with the section modulus at bottom fibre held near the minimum section modulus required. This will keep the minimum eccentricity of the longitudinal prestressing force given by equation (3.9) within the depth of the cross-section.

The active nature of constraints g_3 and g_5 mean that at optimum point, two out of the four stress inequalities given by equation (3.1) to (3.4) are critically satisfied. Constraint g_3 checks the bottom fibre stress at transfer, while g_5 checks the bottom fibre stress at working conditions. Constraint g_5 is active for all designs as the longitudinal prestressing force has been calculated by assuming that the bottom fibre stress at working condition has reached the allowable stress.

The constraints due to minimum thickness of web and the maximum depth of the section are also active. As a combined effect of the two, the tendency of the variables at optimum is to decrease the web thickness and increase the depth. This is because the moment of inertia of such sections are higher.

(b) Hollow Box Section:

In case of Hollow Box section bridges the cost of bridge deck increases with increase in number of longitudinal beams. In the case of 30m span, 5 beam Hollow Box bridge is about 0.792 times the cost of 9 beam bridge deck. For 24m span bridge this ratio is about 0.843. Thus in general for the spans under study 5 beam Hollow Box bridge is economical.

When the cost of Hollow Box bridge is compared with the equivalent T-section bridge in number of longitudinal beams, then the Hollow Box bridge turns out to be uneconomical. In 30m span 5 beam T-section bridge is about 0.875 times the cost of 5 beam Hollow Box section bridge. This ratio is 0.818 for 24m span.

However in certain situations like minimum head room consideration, or when an old bridge is to be replaced and the expence regarding existing road leading to the bridge is to be avoided, Hollow Box section has to be preferred. This is because the overall depth of cross section is less in Hollow Box section when compared with the overall depth of T-section bridges. For example, in case of 30m span bridge the depth of 5 beam Hollow Box section bridge is 0.91 times the depth of 5 beam T-section bridge. This ratio for 24m span bridge is 0.725.

Again the constraint g_5 which checks the bottom fibre stress at working is always active. This once again is

because the pre-stressing force in longitudinal beams has been calculated assuming a section with the bottom stress at working to be critically satisfied. The optimality of the Hollow Box section is governed by the minimum thickness constraints. The minimum thicknesses of top flange, bottom flange and web have been prescribed to be above 14 cms. constraints due to them are active at optimum.

4.5 Few Observations on Numerical Aspects of Optimization Routine

The optimization of penalty function is performed for a successively reducing value of penalty parameter r in SUMT. A large value of r will require too many unconstrained minimization cycles. As suggested by Fox [20] the starting value of r can be taken as

$$r = \frac{OBJ}{\sum_{j=1}^{M} \frac{1}{g_j}} \qquad \ldots 4.1$$

where OBJ is objective function, g_j is j^{th} constraint and M is total number of constraints. They are as defined in Chapter III. In the present work the value of r given by equation 4.1 was very high. By several trials it was found that a starting value of r = 80 with a reduction factor of 0.2 to find the subsequent r, gave satisfactory results. With these values the unconstrained minimization cycles required was four to five. The results given in Table 4.6 are from

computer output when TEEBRI was optimizing 30m span bridge deck consisting of 5 numbers of longitudinal beams. In this only four unconstrained minimization cycles were required to arrive at the optimum point.

Alongwith a reasonable starting value of r, a proper convergence criterion is also required to identify the optimum point and terminate the iterative scheme. T

The D.F.P. algorithm used for unconstrained minimization is terminated whenever any one of the following conditions is satisfied.

(i) $Df^T \cdot \bar{S} \leq \varepsilon_1$; where $\bar{D}f$ is gradient vector and $\bar{S}$ is the move vector.

(ii) $\left| \dfrac{\varphi(\bar{X}_k, r) - \varphi(\bar{X}_{k-1}, r)}{\varphi(\bar{X}_{k-1}, r)} \right| \leq \varepsilon_2$; where k is iteration number of linear search.

(iii) $\max |\bar{D}X| \leq \varepsilon_3$; where $\bar{D}X = \bar{X}_k - \bar{X}_{k-1}$;

$\bar{X}_k$ and $\bar{X}_{k-1}$ are optimum point arrived at after k^{th} and $k-1^{th}$ linear search.

(iv) $Df^T \cdot \bar{D}f \leq \varepsilon_4$

The SUMT algorithm is terminated when both of the following conditions are satisfied:

(i) $\left| \dfrac{\varphi(\bar{X}_i, r_i) - OBJ(\bar{X}_i)}{OBJ(\bar{X}_i)} \right| \leq \varepsilon_5$

where $\bar{X}_i$ and r_i are the value of optimum point and penalty parameter in the i^{th} unconstrained minimization.

(ii) $\max |\overline{DX}| \leq \varepsilon_6$; where $\overline{DX} = \overline{X}_i - \overline{X}_{i-1}$; $\overline{X}_i$ and $\overline{X}_{i-1}$ are optimum point arrived at after i^{th} and $i-1^{th}$ unconstrained minimization cycles respectively.

The value of the small positive numbers taken in the present work are as follows:

$\varepsilon_1 = 0.0005$, $\varepsilon_2 = 0.0001$, $\varepsilon_3 = 0.0005$, $\varepsilon_4 = 0.0001$, $\varepsilon_5 = 0.001$ and $\varepsilon_6 = 0.001$. The effect of these convergence criterion are presented in tables 4.6 and 4.7. The results given in Table 4.7are for the same problem for which the results are shown in Table 4.6.

From Table 4.7 it can be seen that the reduction of penalty function is maximum in the fourth linear search. In the fifth linear search the reduction in penalty function is very low and hence D.F.P. algorithm is terminated. The results of Table 4.6 shows the change of variables to optimum with successively reducing values of r. After end of fourth unconstrained minimization there is no change in design vector from the design vector of 3rd iteration and difference between penalty function and objective function is also very small, hence the program declares the current point as the optimum point.

Table 4.6 : Flow of Design Variables for Different r

Penalty Parameter r	Nos. of Linear search required	Variables of Optimum					Objective Function	Penalty Function	Remarks
		X1	X2	X3	X4	X5			
80.0	5	18.84	16.33	25.69	77.28	193.95	93,364	100,720	Four constraint active
16.0	4	16.89	14.79	23.83	76.67	194.21	86,921	89,406	Five constraint active
3.2	4	16.32	14.36	23.55	76.56	194.20	85,840	86,797	do
0.64	1	16.32	14.36	23.55	76.56	194.20	85,840	86,031	No reduction possible optimum point

Table 4.7 : Change of design variable in different linear search ($r = 80$)

Linear Search iter-ation	Variables at optimum					Objective Function	Penalty Function	Remarks
	X1	X2	X3	X4	X5			
0	20.00	16.00	50.00	90.00	198.00	105.491	118,452	Starting point
1	20.05	15.97	49.98	89.99	194.00	105,433	114,677	
2	21.25	15.67	49.84	89.90	193.95	105,347	112,786	
3	20.98	14.87	48.97	89.33	193.95	105,335	112,416	
4	18.89	16.38	27.27	75.22	193.94	93,577	100752	12 percent reduction
5	18.84	16.36	25.69	77.28	193.95	93,364	100720	Four constraint active

4.6 Conclusions and Recommendations

The cost of bridge deck consisting of either T-sections or Hollow Box sections increases as the number of longitudinal beams increase. T-section bridge is always cheaperr of the two sections and hence its use is recommended. Even though the cost of 7 beam bridge is about 4 percent higher in comparison with the 5 beam bridge deck, use of 7 beam T-section bridge deck is recommended as the modules are lighter by about 23 percent. However, the use of Hollow Box section is recommended when the overall depth of cross section has to be on lower side.

At the time of formulating the optimization problem for minimum cost design of bridge deck the variables have to be properly identified along with right choice of objective function. By over simplification of the formulation of the bridge deck problem and neglecting the cost of cables required for transverse prestressing the sections recommended by the Handbook of S.E.R.C. [9] do not correspond to the minimum cost design.

4.7 Scope for Future Study

In the present work the local effect of wheel load in the top slab has not been accounted for. This, being a secondary stress distribution, it can be superposed on the stress distribution produced in the bridge as a whole to

give the resultant stress distribution. There are several methods available, in the literature, to study these effects. They can be included to check the optimality of this problem.

The minimum cost design of these sections have to be taken up for a wider range of span to have a clear picture of behaviour of optimum point. Other commonly used sections for bridge deck i.e. inverted T-beam (low span range) and composite I-section could also be studied for standardising the minimum cost design of bridge decks.

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C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C      THIS INCORPORATES THE SEQUENTIAL UNCONSTRAINED MINIMIZATION
C      TECHNIQUE
C      SPAN=SPAN OF THE BRIDGE
C      NB= NUMBER OF LONGITUDINAL BEAMS IN THE BRIDGE DECK
C      WLF = WIDTH OF TOP FLANGE OF LONGITUDINAL BEAMS
C      WTF= WIDTH OF TOP FLANGE OF TRANSVERSE BEAMS
C      THETA IS THE FLEXURAL PARAMETER
C      TPL IS ARRAY STORING TRANSVERSE POSITION OF LOAD FOR MAXM.
C      TRANSVERSE BENDING MOMENT
C      PL IS ARRAY STORING POSITION OF LOAD IN LONGITUDINAL DIRECTION
C      FOR MAXM. TRANSVERSE B.M.
C       WP70R IS ARRAY STORING WEIGHTING FACTORS FOR 70R LOADING
C      WPFP IS ARRAY STORING WEIGHTING FACTORS FOR CLASS AA LOADING
C      B IS HALF WIDTH OF BRIDGE DECK
C      BMFLL IS MAXM.B.M. DUE TO LIVE LOAD ON FOOT-PATH AS PER IRC CODE
C      BMFDL IS MAXM. B.M. DUE TO DEAD LOAD OF FOOT-PATH
C      BMWC IS B.M. DUE TO WEARING COARSE
C       BMLL IS B.M. DUE TO 70R/ CLASS-AA LOAD IN BRIDGE SPAN
C      FPTTP IS ALLOWABLE TOP STRESS IN CONCRETE     AT TRANSFER
C      FPTWP IS ALLOWABLE TOP STRESS IN CONCRETE AT WORKING
C      FPBTP IS ALLOWABLE BOTTOM STRESS IN CONCRETE AT WORKING
C       FPBTPIS ALLOWABLE BOTTOM STRESS IN CONCRETE AT TRANSFER
C      SCU IS ALLOWABLE
C      SCU IS 28 DAYS CRUSHING STRENGTH OF CONCRETE
C      CHTW IS COST OF CABLE WITH 12 NOS. OF 7MM WIRES WITH SHEATHING
C      PER CM. RUN
C      CC IS COST OF CONCRETE PER CUBIC CENTIMETRE
C      CAC IS COST OF ONE SET OF ANCHORAGE PER CABLE
C      C1TW IS COST OF ONE WIRE OF 7MM DIA.
C     CS IS COST OF SHEATHING PER CM
C     RE IS LOSS IN PRESTRESSING FORCE
C      X IS ARRAY STORING DESIGN VARIABLE
C      G IS ARRAY STORING THE CONSTRAINTS
C      M IS NUMBER OF CONSTRAINTS , N IS NUMBER OF VARIABLES
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP70R,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
       COMMON/SET3/PT,PO,NTC,NC,APC
       DIMENSION X(10),XD(10),G(15),TPL(4),PL(7),P(7),WP70R(9),WPFP(9),
     1EPSM(4),XS(10)
       DATA TPL/158.0,79.0,0.0,-79.0/
       DATA PL/302.0,698.0,850.0,1063.0,1200.0,1505.0,1642.0/
       DATA P/8000.0,12000.0,12000.0,17000.0,17000.0,17000.0,17000.0/
       DATA WP70R/1.4185,1.7832,0.7194,0.0,0.0,0.0789,0.0,0.0,0.0/
```

```
      DATA WPFP/0.0,0.0,0.0417,0.5833,0.375,0.0,0.0417,0.5833,0.375/
      DATANB,ND,B,RE,FS,CHTW,CAC,C1TW,CS/5,5,550.0,1.25,15000.0,0.18,11
     1.0,0.0091,0.07/
      DATA FPTWP,FPTTP,FPBTP,FPBWP,SCU,CC/110.0,0.01,140.0,0.01,350.0,
     10.000265/
      DATA SPAN,BMLL,WC,FDL/2400.0,0.438335E 08,184.0,591.5/
      DO 5 I=1,4
    5 TPL(I)=TPL(I)/B
      WTF=SPAN/4.0
      FLL=400.0-((40.0*SPAN)/100.0-300.0)/9.0
      FLL=FLL/10000.0
      BMFLL=FLL*SPAN*SPAN/4.0
      BMFDL=FDL*175.0*SPAN*SPAN/40000.0
      BMWC=WC*750.0*SPAN*SPAN/80000.0
      DATA C,IMAX,E,M,N/0.2,10,0.5E-02,15,5/
C
C     M=14,N=4 FOR HOLLOW BOX GIRDER
C
      DATA X/20.0,16.0,40.0,80.0,149.0,0.0,0.0,0.0,0.0,0.0/
C
C     FOR BOX GIRDER X(1)=15.0,X(2)=16.0,X(3)=15.0,X(4)=130.0)
C
      DO 3 I=1,N
    3 XS(I)=X(I)
    7 PRINT 500
  500 FORMAT(5X,*START OF SUMT FOR MINIMUM COST DESIGN OF PRE-STRESSED
     1CONC. BRIDGE*,/,20X,*THE STARTING POINT IS*)
      WLF=2.0*B/FLOAT(NB)
      DO 8 I=1,N
    8 X(I)=XS(I)
      PRINT 510,(I,X(I),I=1,N)
      ITER=0
      PRINT 450,SPAN,NB,SCU,FPTWP,FPTTP,FPBTP,FPBWP
  450 FORMAT(10X,*SPAN IS*,F10.1,2X,*CMS*,/,10X,*NOS.OF BEAM IS*,I3,/
     110X,*CUBE STRENGTH OF CONC. IS*,F8.1,* KG/CM-SQ*,/,5X,*ALLOWABLE S
     2TRESS TOP(WORKING)*,F10.1,/ ,5X,*ALLOWABLE STRESS  TOP(TRANSFER)*,
     4F10.1,/,5X,*ALLOWABLE STRESS BOT.(TRANSFER)*,F10.1,/,5X,*ALLOWABLE
     5 STRESS BOT.(WORKING)*,F10.1)
   11 R=0.0
      CALL FTN(G,X,FI,OBJ,M,N,R)
      R=ABS(OBJ/GKS)
      IF(R.GT.80)  R=80.0
      PRINT 512,R
  510 FORMAT(5(5X,*X(*,I2,*) =*,E16.8/))
  512 FORMAT(////,50X,*R=*,E16.8)
      PRINT  513,(G(I),I=1,M)
      PRINT 515,PT,PO,NTC,NC,APC
  515 FORMAT(2E20.8,2I10,E20.8)
      IF(FI.GT.0.5E 09)  GO TO 1011
      DO 10 I=1,N
```

```
   10 XD(I)=X(I)
      F1=OBJ-R*GKS
      PRINT 514,F1,OBJ
  514 FORMAT(5X,*F =*,E16.8,5X,*OBJ =*,E15.8)
   25 ITER=ITER+1
      CALL UNCON(N,M,X,G,E,TMAX,R)
      CALL FTN( G,X,F,OBJ,M,N,R)
      IF(F.GT.0.5E 09)   GO TO 1001
      E1=ABS(F-F1)/ABS(F)
      E2=0.0
      DO 30 I=1,N
      A=XD(I)-X(I)
      IF(ABS(A).GT.E2)    E2=A
   30 CONTINUE
      IF(E1.LT.0.001.AND.E2.LT.0.001)    GO TO 50
      PRINT 515,PT,PO,NTC,NC,APC
      R=C*R
      PRINT 512,R
      IF(ITER.GT.10)  GO TO 60
      DO 40 I=1,N
   40 XD(I)=X(I)
      F1=F
      GO TO 25
   50 PRINT 605
  605 FORMAT(5X,*CONVERGENCE ACHIEVED*)
   65 PRINT 510,(I,X(I),I=1,N)
      PRINT 513,(G(I),I=1,M)
      PRINT 514,F,OBJ
      GO TO 1001
   60 PRINT 606
  606 FORMAT(5X,*CONVERGENCE NOT ACHIEVED*)
      GO TO 65
  513 FORMAT(5X,*THE CONSTRAINTS ARE*,/9E12.4,/6E12.4)
 1001 NB=NB+1
      IF(NB.GT.9)   GO TO 100
      GO TO 7
 1011 X(3)=X(3)-2.0
      IF(X(3).LT.20.0)  X(3)=20.0
      X(4)=X(4)-2.0
      IK=IK+1
      IF(IK.GT.30)  GO TO 60
      GO TO 11
  100 STOP
      END
      SUBROUTINE FTN(G,X,F,OBJ,M,N,R)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS EVALUATES THE PENALTY FUNCTION(F),OBJECTIVE FUNCTION(OBJ),
C     CONSTRAINTS AT A POINT X FOR A PENALTY PARAMETER R
```

```
C      NOTE THIS SUBROUTINE IS FOR T SECTION
C      SUBROUTINES REQUIRED ARE MAIN,TORSIO,GEN,DISTRI,TRACOF,UNCON,GRAD
C      AND FIBBO
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP7OR,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
      COMMON/SET3/PT,PC,NTC,NC,APC
      DIMENSION X(10),G(15),KO(5,9),KU(5,9),MEU(3,4),MEW(3,4),WP7OR(9),
     1WPFP(9),OK(9,9),ABKO(9),ABKU(9),TPL(4),EU(6),PL(7),P(7)
      REAL KO,KU,MEW,MEU
      HU=X(5)-X(1)-X(3)
      APC=WLF*X(1)+HU*X(2)+X(3)*X(4)
      FMPCB=X(4)*X(3)*X(3)*0.5+X(2)*HU*(X(3)+HU*0.5)+X(1)*WLF*(X(5)-X(1)
     1*0.5)
      YPB=FMPCB/APC
      YPT=X(5)-YPB
      CMI=WLF*(X(1)**3)/12.0+WLF*X(1)*((YPT-X(1)*0.5)**2)+(HU**3)*X(2)/1
     12.0+HU*X(2)*((YPT-X(1)-HU*0.5)**2)+X(4)*(X(3)**3)/12.0+X(4)*X(3)*(
     2(YPB-X(3)*0.5)**2)
      ZPB=CMI/YPB
      ZPT=CMI/YPT
      G(12)=ZPB/ZPT-1.0
      RATIO=WLF/X(1)
      CALL TORSIO(RATIO)
      OI=RATIO*WLF*(X(1)**3)/2.0
      RATIO=HU/X(2)
      CALL TORSIO(RATIO)
      OI=OI+RATIO*HU*(X(2)**3)
      RATIO=X(4)/X(3)
      CALL TORSIO (RATIO)
      OI=OI+RATIO*X(4)*(X(3)**3)
      WTB=25.0
      RATIO=WTF/X(1)
      CALL TORSIO(RATIO)
      OJ=RATIO*WTF*(X(1)**3)/2.0
      HI=X(5)-X(1)-5.0
      RATIO=HI/WTB
      CALL TORSIO(RATIO)
      OJ=OJ+RATIO*HI*(WTB**3)
      ATB=WTF*X(1)+WTB*HI
      TMB=WTB*(HI**2)*0.5+WTF*X(1)*(HI+X(1)*0.5)
      DTB=TMB/ATB
      DTT=X(5)-DTB -5.0
      TJ=WTB*(HI**3)/12.0+WTB*HI*((DTB-HI*0.5)**2)+WTF*(X(1)**3)/12.0+WT
     1F*X(1)*((DTT-X(1)*0.5)**2)
      ZTBB=TJ/DTB
      ZTBT=TJ/DTT
```

```
      SI=CMI/WLF
      SIO=OI/WLF
      SJ=TJ/WTF
      SJO=OJ/WTF
      THETA=B*((SI/SJ)**0.25)/SPAN
      ALPHA=0.2175*(SIO+SJO)/SQRT(SI*SJ)
      SALPH=SQRT(ALPHA)
      CALL DISTRI(THETA,ALPHA,KO,KU)
      CALL GEN(KO,OK)
      DO 10 I=1,9
      XX=0.0
      DO 5 J=1,9
    5 XX=XX+WP70R(J)*OK(I,J)
   10 ABKO(I)=XX/4.0
      CALL GEN(KU,OK)
      DO 20 I=1,9
      XX=0.0
      DO 15 J=1,9
   15 XX=XX+WP70R(J)*OK(I,J)
   20 ABKU(I)=XX/4.0
      Y=0.0
      DO 25 I=1,9
      XX=ABKU(I)-ABKO(I)
      ABKO(I)=ABKO(I)+XX*SALPH
      IF(Y.GT.ABKO(I))  GO TO 25
      Y=ABKO(I)
      IJ=I
   25 CONTINUE
      IM=IJ-1
      DIFB=FLOAT(IM)*B/4.0
      XX=WLF/2.0
      NS=NB-1
      DO 30 I=1,NS
      NBS=I
      Y=DIFB-XX
      IF(Y.LT.WLF)   GO TO 40
   30 XX=XX+WLF
   40 DKW=ABKO(IM)+4.0*Y*(ABKO(IJ)-ABKO(IM))/B
      CALL GEN(KO,OK)
      DO 45 I=1,9
      XX=0.0
      DO 42 J=1,9
   42 XX=XX+WPFP(J)*OK(I,J)
   45 ABKO(I)=XX/2.0
      CALL GEN (KU,OK)
      DO 50 I=1,9
      XX=0.0
      DO 48 J=1,9
   48 XX=XX+WPFP(J)*OK(I,J)
   50 ABKU(I)=XX/2.0
```

```
      DO 55 I=1,9
      XX=ABKU(I)-ABKO(I)
   55 ABKO(I)=ABKO(I)+XX*SALPH
      DKF=ABKO(IM)+4.0*(ABKO(IJ)-ABKO(IM))/B
      DWD=(WLF*(X(5)-5.0)-(APC-X(4)*5.0))*WTB*0.0024
      WS=APC*0.0024
      BMS=WS*SPAN*SPAN/8.0+DWD*SPAN/2.0
      BN=FLOAT(NB)
      PBMFLL=1.1*DKF*BMFLL/BN
      PBMFDL=1.1*DKF*BMFDL/BN
      PBMWC=BMWC/BN
      PBMLL=BMLL*1.1*1.15*DKW/BN
      PBMA=PBMFLL+PBMFDL+PBMWC+PBMLL
      PBMD=BMS+PBMFDL+PBMWC
      PBML=PBMFLL+PBMLL
      ZTP=(PBMA+(1.0-1.0/RE)*BMS)/(FPTWP-FPTTP/RE)
      ZBP=(PBMA+(1.0-1.0/RE)*BMS)/(FPBTP/RE-FPBWP)
      G(1)=ZTP/ZPT-1.0
      G(2)=ZBP/ZPB-1.0
      EP=YPB-0.085*X(5)
      PO=(PBMA+BMS+FPBWP*ZPB)*RE/(EP+ZPB/APC)
      FPBTA=PO/APC+PO*EP/ZPB-BMS/ZPB
      G(3)=FPBTA/FPBTP-1.0
      FPTTA=PO/APC-PO*EP/ZPT+BMS/ZPT
      G(4)=FPTTP/FPTTA-1.0
      FPBWA=(PO/APC+PO*EP/ZPB)/RE -BMS/ZPB-PBMA/ZPB
      G(5)=FPBWP/FPBWA-1.0
      FPTWA=(PO/APC-PO*EP/ZPB)/RE +BMS/ZPT+PBMA/ZPT
      G(6)=FPTWA/FPTWP-1.0
      CN=PO/(0.6*FS*0.385*12.0)
      XX=AMOD(CN,1.0)
      IF(XX.GT.0.0)     GO TO 60
      NC=IFIX(CN)
      ADC=0.0
      Y=0.0
      GO TO 65
   60 Y=1.0-XX
      ADC=XX*12.0
      CN=CN+Y
      NC=IFIX(CN)
   65 ATS=(12.0*(CN-Y)+ADC)*0.385
      EFD=YPT+EP
      UMR=1.5*PBMA+2.5*PBML
      ATSF=0.68*SCU*(WLF-X(2))*X(1)/FS
      ATSW=ATS-ATSF
      G(10)=ATSW*FS/(0.24*0.68*SCU*X(2)*EFD)-1.0
      IF (ATSW.LT.0.0) G(10)=-1.0
      USM=ATSW*FS*EFD*(1.0-0.75*ATSW*FS/(X(2)*EFD*SCU))
      IF(X(1).GE.EFD/4.0)     GO TO 70
      USM=USM+0.7*SCU*X(1)*(WLF-X(2))*(EFD-0.5*X(1))
```

```
   70 G(7)=UMR/USM-1.0
      CALL TRACOE(TPL,MEU,MEW,THETA)
      DO 80 I=1,6
      XX=0.0
      IF(I.GE.4)   GO TO 72
      DO 59 J=1,4
   59 XX=XX+MEW(I,J)
      EU(I)=XX
      GO TO 80
   72 XX=0.0
      IO=I-3
      DO 77 K=1,4
   77 XX=XX+MEU(IO,K)
      EU(I)=XX
   80 CONTINUE
      DO 82 I=1,5,2
      EU(I)=EU(I)+(EU(I+1)-EU(I))*SALPH
   82 EU(I+1)=0.0
      PI=4.0*ATAN(1.0)
      YML=0.0
      IK=0
      DO 90 I=1,5,2
      IK=IK+1
      EI=FLOAT(I)
      XX=0.0
      DO 85 J=1,7
   85 XX=XX+P(J)*SIN(EI*PI*PL(J)/SPAN)
      IJ=IK+1
      YML=EU(I)*XX*((-1.0)**IJ)+YML
   90 CONTINUE
      YML=YML*2.0*B/SPAN
      TYML=YML*WTF
      PT=(TYML+FPBWP*ZTBB+FPTTP*ZTBT/RE)*ATB*RE/(ZTBB+ZTBT)
      G(8)=PT/ATB-1.0
      G( 9)= (ZTBT*PT+RE*TYML)/(RE*ATB*ZTBT*FPTWP)-1.0
      PT=5.0*PT
      TNC=PT/(0.6*FS*0.385*12.0)
      XX=AMOD(TNC,1.0)
      IF(XX.EQ.0.0)    GO TO 100
      Y=1.0-XX
      TNC=TNC+Y
  100 NTC=IFIX(TNC)
      G(11)=15.0*X(5)/SPAN-1.0
      G(13)=13.0/X(1)-1.0
      G(14)=14.0/X(2)-1.0
      G(15)=20.0/X(3)-1.0
      XX=(APC*SPAN+5.0*WTB*(WLF*(X(5)-5.0)-(APC-5.0*X(4))))*CC
      XX=XX+(FLOAT(NC)*CS+(ATS*C1TW/12.0))*SPAN*(1.0+8.0*EP*EP/(3.0*SPAN
     1*SPAN))
      OBJ=XX*BN       +TNC*CHTW*2.0*B
```

```
      OBJ=OBJ+TNC*CAC+CN*CAC*BN
      GKS=0.0
      DO 110 I=1,M
      IF(G(I).EQ.0.0)   GO TO 110
      GKS=GKS+1.0/G(I)
  110 CONTINUE
      F=OBJ-R*GKS
      DO 120 I=1,M
      IF(G(I).GT.0.0.OR.G(I).LT.(-1.0))    F=0.5E 11
  120 CONTINUE
  140 RETURN
      END
      SUBROUTINE TORSIO(RAT)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS GIVES TORSIONAL CONSTANT FOR A RECTANGULAR SECTION
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      B=RAT
      PI=4.0*ATAN(1.0)
      X=0.0
      IF(B.GT.10.0)   GO TO 20
      DO 10 M=1,11,2
      EM=FLOAT(M)
   10 X=X+TANH(EM*PI*B/2.0)  /(EM**5)
      RAT=1.0/3.0-64.0*X/((PI**5)*B)
      GO TO 30
   20 RAT=1.0/3.0
   30 RETURN
      END
      SUBROUTINE TRACOE(TPL,MEU,MEW,A)
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C      THIS GENERATES TWO 3X4 MATRICIES ONE EACH FOR μ0 AND μ1
C
C
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION MEW(3,4),MEU(3,4),TPL(4),AEW(3,5),AEU(3,5)
      REAL MEW,MEU
      E=0.0
      THETA=-A
      PI=4.0*ATAN(1.0)
      B=1.0
      DO 10 I=1,3
      THETA=THETA+2.0*A
      SIG=THETA*PI
      CHG=COSH(SIG)
```

```
SHG=SINH(SIG)
AMB=PI*THETA/(B*SQRT(2.0))
TME=2.0*AMB*B
SHE=SINH(TME)
SNE=SIN(TME)
DO 10 J=1,3
Y=-TPL(J)
CHBPY=COSH(AMB*(B+Y))
CSBPY=COS(AMB*(B+Y))
SNBPY=SIN(AMB*(B+Y))
SHBPY=SINH(AMB*(B+Y))
CSBPE=COS(AMB*(B+E))
CHBME=COSH(AMB*(B-E))
CHBPE=COSH(AMB*(B+E))
CSBME=COS(AMB*(B-E))
SNBPE=SIN(AMB*(B+E))
SHBME=SINH(AMB*(B-E))
SHBPE=SINH(AMB*(B+E))
SNBME=SIN(AMB*(B-E))
EM=1.0/(PI*SQRT(2.0)*THETA*(SHE*SHE-SNE*SNE))
EM1=2.0*SHBPY*SNBPY
TW1=EM1*(SHE*CSBPE*CHBME-SNE*CHBPE*CSBME)
TW2=CHBPY*SNBPY-SHBPY*CSBPY
TW3=SHE*(SNBPE*CHBME-CSBPE*SHBME)
TW3=TW3+SNE*(SHBPE*CSBME-CHBPE*SNBME)
TW=TW2*TW3
AEW(I,J)=EM*(TW1+TW)
U=0.15
OP=1.0+U
OM=1.0-U
TP=3.0+U
PHI=PI*Y/B
SI=PI*E/B
XI=PI-ABS(PHI-SI)
TX=THETA*XI
SHX=SINH(TX)
CHX=COSH(TX)
TSI=THETA*SI
THI=THETA*PHI
CHI=COSH(THI)
SHI=SINH(THI)
CHSI=COSH(TSI)
SHSI=SINH(TSI)
EM=-1.0/(4.0*SIG*SHG*SHG)
TW1=(OM*SIG-TP*SHG)*CHI-(THI*SHG*SHI*OM)
TW1=TW1/(TP*SHG*CHG-OM*SIG)
TW1=TW1*((OM*SIG*CHG-OP*SHG)*CHSI-OM*TSI*SHG*SHSI)
TW2=SIG*CHG*SHI*OM-OM*THI*SHG*CHI
TW2=TW2*((2.0*SHG+SIG*CHG*OM)*SHSI-TSI*SHG*CHSI*OM)
TW2=TW2/(TP*SHG*CHG+OM*SIG)
```

```
      TW=TW1+TW2
      TW=   TW+(OM*SIG*CHG-OP*SHG)*CHX-TX*SHG*SHX*OM
      AEU(I,J)=EM*TW
   10 CONTINUE
      DO 15 I=1,3
      DO 15 J=1,3
      MEW(I,J)=AEW(I,J)
   15 MEU(I,J)=AEU(I,J)
      DO 20 J=1,3
      MEU(J,4)=AEU(J,2)
   20 MEW(J,4)=AEW(J,2)
      RETURN
      END
      SUBROUTINE GEN(A,AA)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS EXPANDS THE KO,AND K1 MATRICIES OF 5X9 TO 9X9 SIZE USING
C     MAXWELL S RECIPROCAL THEOREM
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION A(5,9),AA(9,9)
      DO 10 I=1,5
      DO 10 J=1,9
   10 AA(I,J)=A(I,J)
      DO 20 I=6,9
      DO 20 J=1,9
      IF(J.GE.6)  GO TO 30
      K=I-J
      AA(I,J)=AA(K,I)
      GO TO 20
   30 L=10-I
      K=J-4
      AA(I,J)=AA(K,L)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE DISTRI(A,SUL,KO,KU)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS GENERATES TWO 5X9 MATRICIES , ONE EACH FOR KO AND K1
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION KO(5,9),KU(5,9)
      REAL KO,KU,MEW,MEU
      THETA=A
      PI=4.0*ATAN(1.0)
```

```
      B=1.0
      Y=-1.25*B
      SIG=THETA*PI
      CHG=COSH(SIG)
      SHG=SINH(SIG)
      DO 10 I=1,9
      E=-0.25*B
      Y=Y+0.25*B
      AMB=PI*THETA/(B*SQRT(2.0))
      TME=2.0*AMB*B
      SHE=SINH(TME)
      SNE=SIN(TME)
      DO 10 J=1,5
      E=E+0.25*B
      IF(I.EQ.6.AND.J.EQ.1)     GO TO 100
      IF(I.EQ.7.AND.J.LT.3)     GO TO 100
      IF(I.EQ.8.AND.J.LT.4)   GO TO 100
      IF(I.EQ.9.AND.J.LT.5)     GO TO 100
   30 CONTINUE
      CHBPY=COSH(AMB*(B+Y))
      CSBPY=COS(AMB*(B+Y))
      SNBPY=SIN(AMB*(B+Y))
      SHBPY=SINH(AMB*(B+Y))
      CSBPE=COS(AMB*(B+E))
      CHBME=COSH(AMB*(B-E))
      CHBPE=COSH(AMB*(B+E))
      CSBME=COS(AMB*(B-E))
      SNBPE=SIN(AMB*(B+E))
      SHBME=SINH(AMB*(B-E))
      SHBPE=SINH(AMB*(B+E))
      SNBME=SIN(AMB*(B-E))
      EM=TME/(SHE*SHE-SNE*SNE)
      EM1=2.0*CHBPY*CSBPY
      TW1=EM1*(SHE*CSBPE*CHBME-SNE*CHBPE*CSBME)
      TW2=CHBPY*SNBPY+SHBPY*CSBPY
      TW3=SHE*(SNBPE*CHBME-CSBPE*SHBME)
      TW3=TW3+SNE*(SHBPE*CSBME-CHBPE*SNBME)
      TW=TW2*TW3
      KO(J,I)=EM*(TW1+TW)
      PHI=PI*Y/B
      SI=PI*E/B
      XI=PI-ABS(PHI-SI)
      TX=THETA*XI
      SHX=SINH(TX)
      CHX=COSH(TX)
      TSI=THETA*SI
      THI=THETA*PHI
      CHI=COSH(THI)
      SHI=SINH(THI)
      CHSI=COSH(TSI)
```

```
      SHSI=SINH(TSI)
      RPHI=(SIG*CHG-SHG)*CHI-THI*SHI*SHG
      RSI=(SIG*CHG-SHG)*CHSI-TSI*SHSI*SHG
      QPHI=(2.0*SHG+SIG*CHG)*SHI-THI*SHG*CHI
      QSI=(2.0*SHG+SIG*CHG)*SHSI-TSI*SHG*CHSI
      EM=SIG/(2.0*SHG*SHG)
      KU(J,I)=EM*((SIG*CHG+SHG)*CHX-TX*SHG*SHX+RPHI*RSI/(3.0*SHG*CHG-SIG
     1)+QPHI*QSI/(3.0*SHG*CHG+SIG))
      IF(I.EQ.6.AND.J.EQ.1)    GO TO 110
      IF(I.EQ.7.AND.J.LT.3)    GO TO 110
      IF(I.EQ.8.AND.J.LT.4)    GO TO 110
      IF(I.EQ.9.AND.J.LT.5)   GO TO 110
      GO TO 10
  100 YI=Y
      EI=E
      Y=-Y
      E=-E
      GO TO 30
  110 E=EI
      Y=YI
   10 CONTINUE
      RETURN
      END
      SUBROUTINE UNCON(N,M,X,G,E,IMAX,R)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS INCORPORATES THE D.F.P. ALGORITHM FOR  UNCONSTRAINED MINIMI-
C     -ZATION OF PENALTY FUNCTION F
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP70R,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
      COMMON/SET2/ITER
      COMMON/SET3/PT,PO,NTC,NC,APC
      DIMENSION X(10),G(15),            GRAD(10),S(10),GADP(10),AH(10,10),
     1YQ(10),HY(10)
     2,XD(10)
      DIMENSION TPL(4),PL(7),P(7),WP70R(9),WPFP(9)
 5    CALL FTN(G,X,F,OBJ,M,N,R)
      ITER=1
      TO=0.10
      DO 10 I=1,M
      IF(G(I).GT.0.0)  GO TO 160
   10 CONTINUE
   15 CALL GRADN(N,X,M,R,GRAD)
   20 DO 30 I=1,N
      DO 30 J=1,N
```

```
      AH(I,J)=0.0
      IF(I.EQ.J)        AH(I,J)=1.0
30    CONTINUE
40    SUM=0.
      DO 60 I=1,N
      A=0.0
      DO 50 J=1,N
   50 A=A-AH(I,J)*GRAD(J)
      SUM=SUM+A*A
   60 S(I)=A
      SUM=SQRT(SUM)
      DO 65 I=1,N
   65 S(I)=S(I)/SUM
      IK=0
      A=0.0
      DO 70 I=1,N
      GADP(I)=GRAD(I)
   70 A=A+S(I)*GRAD(I)
      IF(A.GT.0.0)      GO TO 20
      IF(ABS(A).LT.0.5E-03)     GO TO 150
      DO 75 I=1,N
   75 XD(I)=X(I)
      F1=F
      CALL FIBBO(X,N,S,E,IMAX,TO,AL,R,M)
   79 CALL FTN(G,X,F,OBJ,M,N,R)
      CALL GRADN(N,X,M,R,GRAD)
      PRINT 601,ITER
  601 FORMAT(5X,*ITER =*,I3)
      PRINT 602,(X(I),I=1,N)
  602 FORMAT(5X,*VARIABLES ARE*,/,8E15.7)
      PRINT 603,(G(I),I=1,M)
  603 FORMAT(5X,*CONSTRAINT ARE*,/,9E12.4,/,6E12.4)
      PRINT 604,F,OBJ
  604 FORMAT(5X,*F =*,E15.8,*OBJ=*,E15.8)
      PRINT 515,PT,PO,NTC,NC,APC
  515 FORMAT(2E20.8,2I10,E20.8)
      ITER=ITER+1
      IF(ITER.GT.10)  GO TO 150
   77 EU=0.0
      EI=0.
      DO 78 I=1,N
      A=XD(I)-X(I)
      IF(A.GT.EU)     EU=A
      EI=EI+GRAD(I)*GRAD(I)
      XD(I)=X(I)
   78 CONTINUE
      EI=SQRT(EI)
      EO=(F1-F)/F
      IF(ABS(EU).LT.0.005.OR.EI.LT.0.0001.OR.EO.LT.0.0001)   GO TO 150
      F1=F
```

```
      DO 80 I=1,N
   80 YQ(I)=GRAD(I)-GADP(I)
      STY=0.0
      DO 90 I=1,N
   90 STY=STY+S(I)*YQ(I)
      IF(STY.LE.0.1E-15)  GO TO 40
      DO 110 I=1,N
      A=0.0
      DO 100 J=1,N
  100 A=A+AH(I,J)*YQ(J)
  110 HY(I)=A
      YHY=0.0
      DO 120 I=1,N
  120 YHY=YHY+HY(I)*YQ(I)
      DO 130 I=1,N
      DO 130 J=1,N
      AM=S(I)*S(J)/STY*AL
      ON=-HY(I)*HY(J)/YHY
  130 AH(I,J)=AH(I,J)+AM+ON
      GO TO 40
  150 RETURN
  160 PRINT 170,(X(I),I=1,N)
  170 FORMAT(5X,*STARTING POINT VIOLATES CONSTRAINTS*/,8E15.6)
      PRINT 180,(G(I),I=1,M)
  180 FORMAT(9E12.4,/,4E12.4)
      GO TO 150
      END
      SUBROUTINE GRADN(N,X,M,R,GRAD)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS GRADIENT OF THE FUNCTION AT A POINT X USING CENTRAL DIFFERENC
C     SCHEME
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP7OR,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
      DIMENSION TPL(4),PL(7),P(7),WP7OR(9),WPFP(9)
      DIMENSION X(10),G(15),GRAD(10),XP(10),FP(10),FM(10)
      DO 5  I=1,N
    5 XP(I)=X(I)
      DO 10 I=1,N
      XP(I)=X(I)*1.01
      CALL FTN(G,XP,FF,OBJ,M,N,R)
      XP(I)=X(I)
   10 FP(I)=FF
      DO 20 I=1,N
      XP(I)=X(I)*0.99
```

```
      CALL FTN(G, XP,FF,OBJ,M,N,R)
      XP(I)=X(I)
   20 FM(I)=FF
      DO 30 I=1,N
   30 GRAD(I)=(FP(I)-FM(I))/(0.02*X(I))
      RETURN
      END
      SUBROUTINE FIBBO(X,N,S,E,IMAX,TO,AL,R,M)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS FINDS THE OPTIMAL STEP LENGTH IN DIRECTION S BY FIBONACCI
C     METHOD
C
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP7OR,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
      COMMON/SET2/ITER
      DIMENSION X(10),G(15),FI(20),TPL(4),PL(7),P(7),WP7OR(9),WPFP(9),
     1XA(10),XB(10),S(10)
      AL=0.0
      KJ=0
      STEP=0.0
      IMAX=15
      K=0
      IF(R.GT.5.0)   TO=1.0/FLOAT(ITER)
      IF(R.LT.5.0)   TO=0.5/FLOAT(ITER)
      DO 5 I=1,N
    5 XA(I)=X(I)
      CALL FTN(G,XA,FA,OBJ,M,N,R)
   10 DO 15 I=1,N
   15 XB(I)=XA(I)+TO*S(I)
      CALL FTN(G,XB,FB,OBJ,M,N, R)
      IF(FB.LE.FA)  GO TO 20
      GO TO 40
   20 DO 25 I=1,N
   25 XA(I)=XB(I)
      STEP=STEP+TO
      FA=FB
      K=K+I
      GO TO 10
   40 A1=0.0
      B1=TO
      DO 41 I=1,N
   41 X(I)=XA(I)
   42 FI(1)=1.0
      FI(2)=1.0
      DO 45 I=3,IMAX
```

```
 45 FI(I)=FI(I-2)+FI(I-1)
    II=IMAX
    AL2S=FI(II-2)/FI(II)*(B1-A1)
    J=2
900 AL1=B1-A1
    IF(AL2S.GT.AL1/2.0)   GO TO 300
    IF(AL2S.EQ.AL1/2.0)   GO TO 111
    X1=AL2S+A1
    X2=B1-AL2S
    GO TO 400
111 X1=B1-AL2S-0.005
    X2=A1+AL2S+0.005
    GO TO 400
300 X1=B1-AL2S
    X2=AL2S+A1
400 DO 401 I=1,N
    XB(I)=X(I)+X1*S(I)
401 XA(I)=X(I)+X2*S(I)
    CALL FTN(G,XA,F1,OBJ,M,N,R)
    CALL FTN(G,XB,F2,OBJ,M,N,R)
    IF(F1-F2)   600,450,500
450 A1=0.0
    B1=X1
    IF(J.EQ.(N-1))  GO TO 700
    N1=N-J-1
    N2=N-J
    AL2S=FI(N1)/FI(N2)*(B1-A1)
    J=J+1
    GO TO 700
500 A1=X1
    AL2S=X2-A1
    GO TO 700
600 B1=X2
    AL2S=X1-A1
700 J=J+1
    IF(J.EQ.N)    GO TO 800
    GO TO 900
800 STEP=STEP+A1
    AL=AL+STEP
    RETURN
    END
    SUBROUTINE FTN(G,X,F,OBJ,M,N,R)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     THIS FINDS THE PENALTY FUNCTION, OBJECTIVE FUNCTION AND CONSTRAIN
C    TS FOR BOX TYPE SECTION AT A DESIGN POINT X
C    SUBROUTINES REQUIRED ARE MAIN,GEN,DISTRI,TRACDE,UNCON,GRADN AND
C    FIBBO
C
```

```
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON/SET1/SPAN,WTF,WLF,NB,THETA,TPL,PL,P,WP7OR,WPFP,B,BMFLL,BMFD
     1L,BMWC,BMLL,FPTWP,FPTTP,FPBTP,FPBWP,SCU,FS,CC,CHTW,CAC,GKS,RE
     2,C1TW,CS
      COMMON/SET3/PT,PO,NTC,NC,APC
      DIMENSION X(10),G(15),KO(5,9),KU(5,9),MEU(3,4),MEW(3,4),WP7OR(9),
     1WPFP(9),OK(9,9),ABKO(9),ABKU(9),TPL(4),EU(6),PL(7),P(7)
      REAL KO,KU,MEW,MEU
      WTB=25.0
      HU=X(4)-X(1)-X(2)
      APC=WLF*(X(1)+X(2))+HU*2.0*X(3)
      FMPCB=WLF*(X(2)**2)*0.5+HU*X(3)*(X(2)+HU*0.5)*2.0+WLF*X(1)*(X(4)-0
     1.5*X(1))
      YPB=FMPCB/APC
      YPT=X(4)-YPB
      CMI=WLF*(X(1)**3)/12.0+WLF*X(1)*((YPT-0.5*X(1))**2)+2.0*X(3)*(HU**
     13)/12.0+2.0*X(3)*HU*((YPT-HU*0.5-X(1))**2)+X(2)**3*WLF/12.0+WLF*X(
     22)*((YPB-0.5*X(2))**2)
      ZPB=CMI/YPB
      ZPT=CMI/YPT
      HA=HU*(WLF-2.0*X(3))
      OI=4.0*HA*HA/((WLF-2.0*X(3))/X(1)+(WLF-2.0*X(3))/X(2)+2.0*HU/X(3))
      TJ=WTF*(X(4)**3)/12.0-(WTF-2.0*X(3))*(HU**3)/12.0
      HT=(WTF-2.0*X(3))*HU
      OJ=4.0*HT/((WTF-2.0*X(3))/X(1)+(WTF-2.0*X(3))/X(2)+2.0*HU/X(3))
      TMB=WTF*(X(2)**2)*0.5+HU*X(3)*(X(2)+HU*0.5)*2.0+WTF*X(1)*(X(4)-0.5
     1*X(1))
      ATB=WTF*(X(1)+X(2))+HU*2.0*X(3)
      DTB=TMB/ATB
      DTT=X(4)-DTB
      ZTBB=TJ/DTB
      ZTBT=TJ/DTT
      SI=CMI/WLF
      SIO=OI/WLF
      SJ=TJ/WTF
      SJO=OJ/WTF
      THETA=B*((SI/SJ)**0.25)/SPAN
      ALPHA=0.2175*(SIO+SJO)/SQRT(SI*SJ)
      SALPH=SQRT(ALPHA)
      CALL DISTRI(THETA,ALPHA,KO,KU)
      CALL GEN(KO,OK)
      DO 10 I=1,9
      XX=0.0
      DO 5 J=1,9
    5 XX=XX+WP7OR(J)*OK(I,J)
   10 ABKO(I)=XX/4.0
      CALL GEN(KU,OK)
      DO 20 I=1,9
      XX=0.0
```

```
      DO 15 J=1,9
   15 XX=XX+WP7OR(J)*OK(I,J)
   20 ABKU(I)=XX/4.0
      Y=0.0
      DO 25 I=1,9
      XX=ABKU(I)-ABKO(I)
      ABKO(I)=ABKO(I)+XX*SALPH
      IF(Y.GT.ABKO(I))  GO TO 25
      Y=ABKO(I)
      IJ=I
   25 CONTINUE
      IM=IJ-1
      DIFB=FLOAT(IM)*B/4.0
      XX=WLF/2.0
      NS=NB-1
      DO 30 I=1,NS
      NBS=I
      Y=DIFB-XX
      IF(Y.LT.WLF)   GO TO 40
   30 XX=XX+WLF
   40 DKW=ABKO(IM)+4.0*Y*(ABKO(IJ)-ABKO(IM))/B
      CALL GEN(KO,OK)
      DO 45 I=1,9
      XX=0.0
      DO 42 J=1,9
   42 XX=XX+WPFP(J)*OK(I,J)
   45 ABKO(I)=XX/2.0
      CALL GEN (KU,OK)
      DO 50 I=1,9
      XX=0.0
      DO 48 J=1,9
   48 XX=XX+WPFP(J)*OK(I,J)
   50 ABKU(I)=XX/2.0
      DO 55 I=1,9
      XX=ABKU(I)-ABKO(I)
   55 ABKO(I)=ABKO(I)+XX*SALPH
      DKF=ABKO(IM)+4.0*(ABKO(IJ)-ABKO(IM))/B
      DWD=(WLF*X(4)-APC)*WTB*0.0024
      WS=APC*0.0024
      BMS=WS*SPAN*SPAN/8.0+DWD*SPAN/2.0
      BN=FLOAT(NB)
      PBMFLL=1.1*DKF*BMFLL/BN
      PBMFDL=1.1*DKF*BMFDL/BN
      PBMWC=BMWC/BN
      PBMLL=BMLL*1.1*1.15*DKW/BN
      PBMA=PBMFLL+PBMFDL+PBMWC+PBMLL
      PBMD=BMS+PBMFDL+PBMWC
      PBML=PBMFLL+PBMLL
      ZTP=(PBMA+(1.0-1.0/RE)*BMS)/(FPTWP-FPTTP/RE)
      ZBP=(PBMA+(1.0-1.0/RE)*BMS)/(FPBTP/RE-FPBWP)
```

```
      G(1)=ZTP/ZPT-1.0
      G(2)=ZBP/ZPB-1.0
      EP=YPB-0.085*X(4)
      PO=(PBMA+BMS+FPBWP*ZPB)*RE/(EP+ZPB/APC)
      FPBTA=PO/APC+PO*EP/ZPB-BMS/ZPB
      G(3)=FPBTA/FPBTP-1.0
      FPTTA=PO/APC-PO*EP/ZPT+BMS/ZPT
      G(4)=FPTTP/FPTTA-1.0
      FPBWA=(PO/APC+PO*EP/ZPB)/RE -BMS/ZPB-PBMA/ZPB
      G(5)=FPBWP/FPBWA-1.0
      IF(ABS(G(5)).LT.0.001)      G(5) =0.0
      FPTWA=(PO/APC-PO*EP/ZPB)/RE +BMS/ZPT+PBMA/ZPT
      G(6)=FPTWA/FPTWP-1.0
      CN=PC/(0.6*FS*0.385*12.0)
      XX=AMOD(CN,1.0)
      IF(XX.GT.0.0)     GO TO 60
      NC=IFIX(CN)
      Y=0.0
      ADC=0.0
      GO TO 65
   60 Y=1.0-XX
      ADC=12.0*XX
      CN=CN+Y
      NC=IFIX(CN)
   65 ATS=((CN-Y)*12.0+ADC)*0.385
      EFD=YPT+EP
      UMR=1.5*PBMA+2.5*PBML
      ATSF=0.68*SCU*(WLF-X(2))*X(1)/FS
      ATSW=ATS-ATSF
      G(10)=ATSW*FS/(WLF*EFD*SCU*0.24)-1.0
      USM=ATSW*FS*EFD*(1.0-0.75*ATSW*FS/(X(2)*EFD*SCU))
      IF(X(1).GE.EFD/4.0)     GO TO 70
      USM=USM+0.7*SCU*X(1)*(WLF-X(2))*(EFD-0.5*X(1))
   70 G(7)=UMR/USM-1.0
      CALL TRACOE(TPL,MEU,MEW,THETA)
      DO 80 I=1,6
      XX=0.0
      IF(I.GE.4)   GO TO 72
      DO 59 J=1,4
   59 XX=XX+MEW(I,J)
      EU(I)=XX
      GO TO 80
   72 XX=0.0
      IO=I-3
      DO 77 K=1,4
   77 XX=XX+MEU(IO,K)
      EU(I)=XX
   80 CONTINUE
      DO 82 I=1,5,2
      EU(I)=EU(I)+(EU(I+1)-EU(I))*SALPH
```

```
 82 EU(I+1)=0.0
    PI=4.0*ATAN(1.0)
    YML=0.0
    IK=0
    DO 90 I=1,5,2
    IK=IK+1
    EI=FLOAT(I)
    XX=0.0
    DO 85 J=1,7
 85 XX=XX+P(J)*SIN(EI*PI*PL(J)/SPAN)
    IJ=IK+1
    YML=EU(I)*XX*((-1.0)**IJ)+YML
 90 CONTINUE
    YML=YML*2.0*B/SPAN
    TYML=YML*WTF
    PT=(TYML+FPBWP*ZTBB+FPTTP*ZTBT/RE)*ATB*RE/(ZTBB+ZTBT)
    G(8)=PT/ATB-1.0
    G( 9)= (ZTBT*PT+RE*TYML)/(RE*ATB*ZTBT*FPTWP)-1.0
    PT=5.0*PT
    TNC=PT/(0.6*FS*0.385*12.0)
    XX=AMOD(TNC,1.0)
    IF(XX.EQ.0.0)    GO TO 100
    Y=1.0-XX
    TNC=TNC+Y
100 NTC=IFIX(TNC)
    G(11)=15.0*X(4)/SPAN-1.0
    G(12)=14.0/X(1)-1.0
    G(13)=14.0/X(2)-1.0
    G(14)=14.0/X(3)-1.0
    XX=(APC*SPAN+(X(4)-X(1)-X(2))*(WLF-X(3)*2.0)*5.0*WTB)*CC
    XX=XX+(FLOAT(NC)*CS+(ATS*C1TW/12.0))*SPAN*(1.+8.*EP*EP/(3.*SPAN*SP
   1AN))
    OBJ=XX*BN+TNC*CHTW*2.0*B
    OBJ=OBJ+TNC*CAC+CN*CAC*BN
    GKS=0.0
    DO 110 I=1,M
    IF(G(I).EQ.0.0)   GO TO 110
    GKS=GKS+1.0/G(I)
110 CONTINUE
    F=OBJ-R*GKS
    DO 119 I=1,M
    IF(G(I).GT.0.0)       F=0.5E 11
119 CONTINUE
    RETURN
    END
```